

## **Tactical Ballistic Missile (TBM) Composite Tracking Protocols for Single Integrated Air Picture(SIAP) Optimized Attributes Risk Constrained by Discrimination, Classification, Architecture and Data Registration, Army System of Systems (ASoS) and Integrated Air Missile Defense (IAMD)**

**Abstract** *Alternative Composite Tracking Protocols for a Composite Track File will be achieved by the theoretical performance evaluation of two classes of software theory protocols on a Network Centric Topology. The Network Centric Topology for theoretical performance evaluation of the two classes of software theory protocols, Composite Network Centric Fractal/Graphic (CTNCF/G) Protocol and Composite Network Centric Holographic (CTNCH) Protocol, will be comprised of a STAR Topology with a Simulation Facility center, i.e. Army GIG through the CIA, and the multiple STAR facets comprised of gateway software linking the Simulation Facility and the Sensors. The optimization of the Single Integrated Air Picture(SIAP) Attributes with the following constraints, the Data Registration concepts, the Army Integrated Air and Missile Defense (AIMD) System of Systems (SoS) architecture, discrimination and classification parameters will be reflected along with the perturbation sensitivity theory in the two theoretical software protocol classes representing solutions for the imaging Composite Track Files. The key technical risks and programmatic risks are developed for the composite tracking capability.*

**Keywords:** *Network Centric Topology, (CTNCF/G) Protocol, CTNCH Protocol, Star Topology, AIMD, SIAP, Composite Tracking Protocols*

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### **1. Introduction**

The basic definition of a Composite Track File concept for multiple sensor track sources and performing discrimination function from an imaging definition is an entity made up of various parts and elements especially containing recognizable constituents. A composite image is a picture made up of several parts or elements, i.e. the composite order of architecture. Within the composite image, the sections can be created by juxtaposition or overlaying, realizing a contrasting effect. The Composite Tracker will be processing and correlating aircraft and high velocity missile tracks from multiple military sensors representing a threat engagement. The TBM (Tactical Ballistic Missile) correlation/association process at the sensors generates this associated measure/tracklet/track report. The Composite Track File concept must fuse the measurement tracklet or track state from multiple sensors into a single, integrated Composite Track of the object. The demonstration of eliminating redundant tracks includes the critical component Data Registration in the Composite Track File definition.

Data Registration is the process of correcting for navigation, alignment and in the presence of maneuvering targets or closely spaced objects, correct data association establishing the correct

number of tracks for the correct number of truth objects, when the individual objects are resolvable by the sensors. Data association also ensures that the correct identification data for air breathing targets and correct discrimination data for ballistic missiles gets paired with the correct tracks. Classification, Discrimination and Identification(CDI), a critical conceptual component of the air picture requires sensor resources allocation with the integrated knowledge that the other sensors are present.

The Integrate Air and Missile Defense Battle Command System(IBCS) Composite Tracking function will adhere to SIAP Attributes: completeness, clarity, continuity, kinematics accuracy, ID-Completeness, ID correctness, ID-Ciarity, and commonality. The technical issues for Composite Track capability processes are 1) situations in which track classification is ambiguous, 2) allowing track classification decisions that require correction, 3) distributed track data function, 4) track related data interaction with the battle management function, 5) host system displays, 6) allocation between IABM and ASoS sensor component and 7) identification of interfaces between systems distributed system components.

Classification and discrimination of air breathing target, TBMs and associated objects that support IBCS fire control timelines, enables the component sensors within the ASoS JAM[I ,20) network to vary in terms of operating band detection range and bandwidth. The identification of the lethal or potential lethal objects among the inbound objects requires component discrimination to detect lethal objects based on features from multiple sensors and during multiple phase of TBM flight by combining the sensor data from ASoS sensors with the data from MDS sensors and to refine the discrimination solutions. The Composite Tracking protocols must be integrated into the SIAP Joint Track Manager. The potential sensors to be in the ASoS architecture are ICENS, Patriot radar, Sentinel radar and MDA ANTPY-2 class radar (e.g. THAAD, Forward Based). The ASoS performs Composite Tracking on TBM and support Composite Tracks and into the joint SIAP. The Component Tracking will be in a distributed manner aros these above referenced sensor systems for both TBMs and ABTs simultaneously.

## **2. ASoS Network Centric Architecture**

The ASoS architecture will be Network Centric where component sensor and weapon elements can serve the netted architecture. The ASoS architecture will integrate sensors, weapons and a common battle command element across a single integrated Fire Control (FC) Network Common Battle Command Element as the Integrated Air and Missile Defense Battle Command System (iBCS). The optical Composite Track File Protocol will realize imaging, holography, compression and expansion theoretical concepts to achieve the above referenced data registration, SIAP Attributes, sensor requirements and ASoS architecture.

### **2.1. Composite Track File Software Protocol Network Centric Performance Evaluation**

To investigate and evaluate alternative Composite Tracking and discrimination techniques and architectures from multiple individual Sensor Track sources and perform a discrimination function based on a Composite Track File requires the theoretical development of two classes of Composite Tracking Software Protocols, Composite Tracking Network Centric Fractal/Graphic (CTNCF/G) Protocol[4,5,11] and the Composite Tracking Network Centric Holographic (CTNCH)

Protocol[1,6,7,12]. The Composite Tracking File is obtained from a process where several different source images are merged and manipulated for theoretical performance evaluation of two classes of optics based software protocols proposed to perform this imaging study on a Network Centric Topology. Refer to Figure 1 for the Network Centric Topology and Figure 2 for the Geometric Software Structure for Twelve Sensor Composite Tracks.

The TAMD CRD related SIAP Attributes[21] Objective Functions Optimization[3,18] will be achieved with the constraint functions for ASoS Classification, ASoS Discrimination, ASoS Architecture and Data Registration. To prevent redundant track constraints the linkage of the measurement tracklet or track data from multiple sensors into a single integrated Composite Track of the object will be integrated into the rules associated with objects to report. At the center of the Network Centric Star Topology is the Simulation Facility (SF) linked to the twelve Sensors at the ends of the twelve facets of the Star Topology by twelve Gateway Software (GS) links (GW1-GW12). This main SF is also linked to the Army GIG(Land War Net) to obtain CIA Security information and intelligence[14,18). Each of the twelve Gateway Software links connect the twelve Sensors with the twelve individual simulation facilities (SFI-SF12). The twelve individual simulation facilities then communicate with the Network Centric Main Simulation Facilities (SF), which oversees their individual information problems. The sensors can be included in LAN Networks.

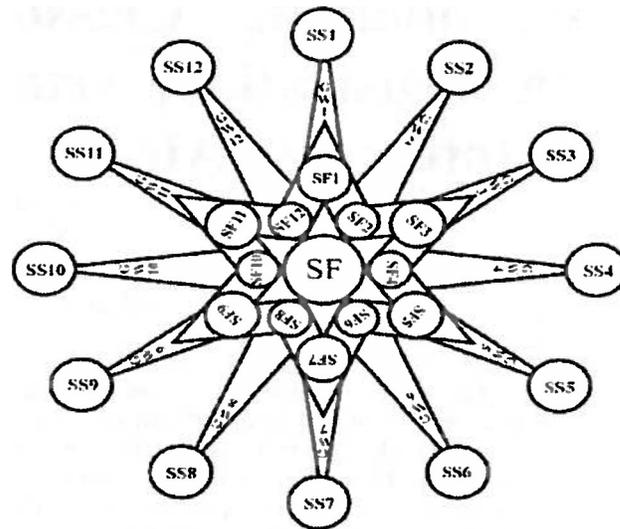


Figure I. Network Centric Topology for Composite Track File Software Performance Evaluation  
SF-Main Centralized Simulation Facility SS-Satellite Sensors (1-1)

### SFI-SFJ2 – Individual GW-Gateway Simulation Facility for Software Star Facets each Satellite Sensor (1-12)

The ASoS Architecture[20] parameter constraint integrates sensor weapons and a common battle command element across a single Integrated File Control (IFC) Network. The Army Integrated and

Missile Defense (AIAMD) ASoS also enables increased operational flexibility to meet the needs of the current and future battlefield. The ASoS Architecture will be performance evaluated on a Network Centric Topology for multiple sensors. The ASoS Architecture evolves into a Network Centric Architecture, where component sensor and weapon elements can serve the netted architecture instead of individual command and control elements. The Network Centric Architecture allows deployment and employment of Mission, Enemy, Terrain and, Weather, Troops and support available, time available, and civil consideration (METT-TC) tailored forces of mixed sensors and weapons that are all controlled through the ASoS Common C2. The ASoS Architecture will provide backward capability to enable non- modified systems to interoperate with a deployed task force. ASoS will enable weapons and sensors to operate fused data. The ASoS Architecture and Common systems will enable dynamic defense design and task force reorganization, a condition on the Terrorist space to change while continuing on- going tactical operation. The ASoS Common C2 provides the capability for interdependent Network Centric Operations that link Joint Integrated Air and Missile Defense (IAMD) protection to the supported forces change of operation and maneuvers. The mathematical operation that converts a two dimensional image into a holographic image is a conformal mapping transformation [1,9]. The ASoS contributions to three dimensional SAISU includes actions and capabilities providing visualization and understanding pictures of activities or events occurring in this dimension. Situational awareness encompasses seeing and knowing the air space and the objects that fly or are flying in the airspace as part of a Common Operational Picture (COP)/Common Tactical Picture (CTP).

### 3. Composite Tracking Network Centric Software Protocols

#### 3.1. Composite Tracking Network Centric Fractal Graphic(CTNCF/G) Software Protocol Theory

The software theory for the CTNCF/G Protocolis represented by detenninistic geometry to describe specific objects, i.e. tracks and structure, with equations so the model can be reproduced. Two features of computer graphic modeling are computer geometric structure and distributions of color and brightness scales and hierarchical layout of objects. Affine transfromlations, scalings, rotations and contingencies define structures. Fractal sets contain infinitely many points defined by the relations between these pieces. Fractal geometry extends classical geometry to deterministic fractals. Image compression in the digitization of images is represented by the mathematical definition of the Hyperbolic Interated Function System (HIFS) with probabilistic properties of Collage Theory utilizing the Markov property [4] equation (1):

$$\text{HIFS} = \langle \mathbf{K}(\mathbf{X}, \mathbf{h}(\mathbf{d})), \mathbf{P}(\mathbf{x}), (\mathbf{X}, \mathbf{d}), \mathbf{w}, \mathbf{A}, \mathbf{s}, \mathbf{d}_H, \mathbf{M}, \boldsymbol{\mu}, \mathbf{C} \rangle$$

$$= \{ \mathbf{X}: \mathbf{w}_n (n=1, 2, \dots, N), \mathbf{w}: \mathbf{K}(\mathbf{x}) \rightarrow \mathbf{K}(\mathbf{x}) \} \quad (1)$$

**IFS** = fractal framework for dynamical systems, i.e. fractal fixed points of certain settings;  
**K** = metric space whose elements are nonempty compact sets of a space like  $\mathbb{R}^2$ ,

**h(d)** = Hausdorff Metric illustrates that d is the Hausdorff distance between points A and B in  $\mathbf{K}(\mathbf{X})$ ,

$\mathbf{K}(\mathbf{X}, \mathbf{h}(\mathbf{d}))$  = the space of fractals, any subset of which is a fractal, a complete metric space,

$\mathbf{P}(\mathbf{X})$  = space of normalized Borel Measures on a compact metric space,

$\mathbf{w}$  = a contraction mapping with contractivity factor  $s$ ,

$\mathbf{A}$  = Attractor of the IFS,  $s$  = contractivity factor,  $\text{MAX}(s_n = n = 1, 2, \dots, N)$ ,

$\mathbf{d}_H$  = Hutchinson distance between measures degree that two images look alike, with the same number of points,

$\mu$  = invariant measure appearing like a given texture when represented by a set of dots;

$\mathbf{M}$  = Markov operators for HIFS with probabilities,

$\mathbf{C}$  = Condensation transformation, a contraction mapping with contractivity factor zero. Refer to Figure 2 for the flow chart of the CTNCF/G Protocol.

### 3.2. Composite Tracking Network Centric Holographic (CTNCH) Software Protocol Theory

The basic software theory for the Composite Tracking Network Holographic (CTNCH) Protocol contains the following three mathematical concepts to holographically represent the Terroristspace: (1) Fourier Transform, Discrete Fourier Transform (DFT)[6], (2) invariance, perturbation and sensitivity theory[15] and (3) conformal mapping[9] for deterministic and probabilistic imaging. The two and three dimensional holographic equations are a combination of the DFT and the Modulation Transfer Function {MTF}[6,15] representing the frequency characteristics of the lenses, filters, and imaging. The two and three dimensional holographic equations are a combination of the DFT and the Modulation Transfer Function (MTF) representing the frequency characteristics of the lenses, filters, and mirrors and perturbation sensitivity equations for the Terroristspace vibration and other image transforming and uncontrolled alteration.

The term invariance in computer graphic fractal theory[4] for the HIFS mathematical formalism is analogous to the holographic invariance in the Control Theory formalism for the perturbation and sensitivity theory. The Fourier Hologram begins mathematically with the DFT of the image that will be altered by optical devices to form the perturbed three dimensional holographic image [6, 7, 12, 13]:

$$F\{g(x,y)\} = \sum_{x=1}^n g(x,y) \sum_{y=1}^n \exp 2\pi i [(f_x - 1)(x-1)/N] + [(f_y - 1)(y-1)/N] \quad (2)$$

The convolution of the object image and the mirror or lens MTF, enabling the holographic representation of the image is obtained by the product of the lens MTF and the frequency representation of the object image [5, 14, 25]:

$$F\{g(x,y) * h(x,y)\} = G(f_x, f_y) * H(f_x, f_y) \quad (3)$$

Figure 2 CTNCF/G Protocol Flow Chart Characteristics

Conformal mapping [7] in holographic optics is described by the placement of the optical components to enable the three dimensional representation of the object and its correlated image

described by the Fourier DFT. Conformal Mapping theory, also referenced as the Conformal Mapping Transformation by Born and Wolf [1,2] enables placement of the optical components for the three dimensional holographic effect by arranging the optical lenses, filters, mirrors and optical memory devices at various angles to each other. Refer to Figure 4 for the flow chart of the CTNCH Protocol.

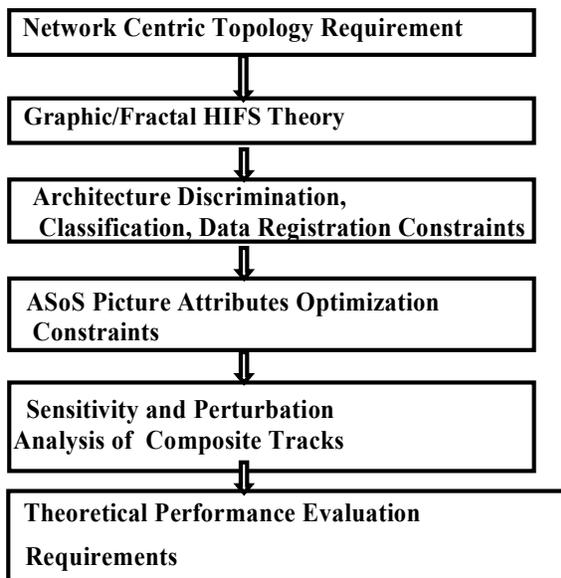


Figure 2 CTNCF/G ProtocolFlow Chart Characteristics

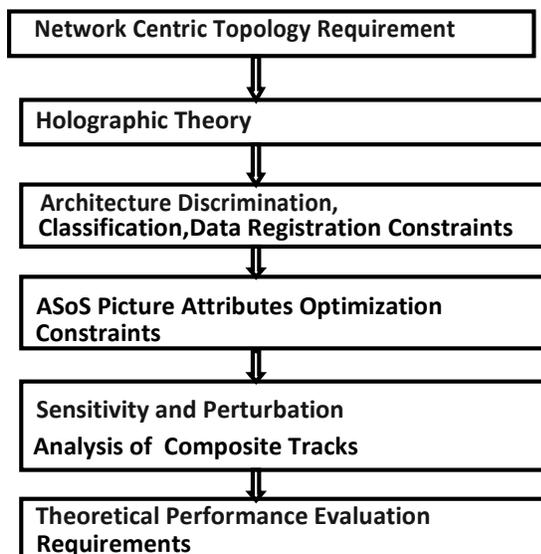


Figure 3 CTNCH Protocol Flow Chart Characteristics

### 3.3. Perturbation Sensitivity Analysis for Both Protocol Classes

The perturbation sensitivity analysis with the DFT representation of the Hologram requires the following equation for  $R(t, u_0)$  the solution of a differential equation for  $R(t, u_0)$  the solution of a differential equation describing a dynamic system,

$$R(t, u_0 + \Delta u) = R(t, u_0) + \frac{\partial R}{\partial u_0} \Delta u + \dots \quad (4)$$

The total resolution sensitivity function for the system yielding Terroristspace vibration is;

### 4. Lagrangian Optimization Of SIAP Attributes

The composite tracking capability will be performed in a distributed fashion across the ASoS referenced systems for both TBM's and ABT's [20] simultaneously. The SIAP KPP's [20] attributes and metrics for air breathing objects as defined in the TAMD CRD are optimized for the four constraints of Architecture, Classification, Discrimination and Data Registration, because the determination of composite track and tracklet imaging requires all of the information concerning the attributes until the air breathing missiles or threats are intercepted. The Directional Derivative and the Game Theory, according to Danskin [3] have the common goal of determining the maximum point where the end of a Game occurs, i.e. the point of the removal of all constraints in a marginal or Lagrangian equation. The change from constrained optimization, i.e. unconstrained optimization, at the end of a Game allows the loss of information accumulated during the game. Constraint removal occurs with the interception of a missile. Separable removal of constraints is offered at the end of a stochastic Game or the sequence of interception and firing events. The SIAP Attribute Optimization for the four constraints is required to obtain the composite track because it doesn't require the destruction of the objects.

During the Strategic Defense Initiative Organization Project [15, 20] constraints impacting the interception of an ICBM Missile by the TMD Kinetic Kill Vehicle were stated as the constraints on the missile as the number of camouflaging balloons or other camouflaging devices. The Constraints on the Kinetic Kill Vehicle [18] were: (1) the firing sequence events, (2) the varying distance of the impact with the missile, and (3) the projected speed differences. These constraints are considered in a general manner in the CTNCF/G Protocol Classes and the CTNCH Protocol Classes software theory in equations 8-15 of the optimized SIAP Attributes. The composite track objects are not destroyed like the missiles.

#### 4.1. Technical Risks And Programmatic Risks For SIAP Attribute Constraints

Key technical risks are developed for a composite tracking capability and data association between multiple sensor sources and data registration. Data association deals with corrections for pairing of track data with an object. Data Registration allows for correction of navigation, alignment and timing errors across multiple sensor platforms. Risk is

$$\Delta R = \sum_{i=1}^n \frac{\partial R}{\partial u_{oi}} \Delta u_i \quad (5)$$

defined as the chances that a particular decision or action can

$\frac{\partial R}{\partial u_{oi}}$  = the sensitivity influence coefficient about the operating point of each component parameter.

$\Delta u_i$  = value of the respective parameter tolerance.

The solution is geometric rather than algebraic when the tolerances are treated as statistical random disturbances. For statistically dependent compositions dependent on the values of those in a previous subsystem the law of cosines should be used to compose the **R** vectors for each component. The algebraic solution is a method of directly calculating the sensitivity function for the maximum value when the tolerances are not random during the process or for a worse case.

give rise to a variety of outcomes for which the mathematical probability can be calculated. Therefore,

the programmatic risk is the integration of the composite tracking protocols into the SIAP Joint Track Manager. Risk is defined economically by the following equation[19]:

$$\mathbf{Risk}=\mathbf{T}\mathbf{x}\mathbf{V}\mathbf{x}\mathbf{C} \quad (6)$$

T= threat, the frequency of potentially adverse events, and protection of goals.

V = vulnerability, the likelihood of success of give rise to a variety of outcomes for which the mathematical a particular threat category against a particular organization.

C = cost is the total impact of a particular threat exercised by a vulnerable target.

The mathematical probabilistic mitigating risk equation is the following[8]:

$$\mathbf{Risk}=(\mathbf{P}\mathbf{a})(\mathbf{I}-\mathbf{P}\mathbf{e})(\mathbf{C}) \quad (7)$$

**Pa**= the probability of attack from the analysis of threat based

on intelligence, history, capabilities, intentions, targeting, existence of the threat, current security environment and other information to arrive at some indications of an event, at worst case =1.0. Use a value for likelihood of attack Pa other than the assumed worst case value of 1.0 to be used to help discriminate among the target set. Pe =system effectiveness is the product of Pi and Pn. **Pi** is the probability of interruption indicating how effective the protective system is in interrupting an adversary attack.**Pn** = the probability of neutralization, how well response measures do in force-on-force conflicts with the adversary given interruption. C = Consequence of an event including prioritized targets to include mission importance, criticality and impact from blast chemical biological agents and radiological sources. It will link consequences to protection upgrades and mitigation decisions and determine the effect of such changes.

#### 4.2 Composite Tracking Optimization Of SIAP Attribute Constraints Description

The Lagrangian Optimization Equations for the SIAP Attributes [20] required for the CTNCF/G Protocol and

CTNCH Protocol have Objective Function and ASoS Classification, ASoS Discrimination ASoS Architecture, and Data Registration constraints listed in equations (8)-(15) for each SIAP Attribute Objective Function. Each of the four constraints for each attribute will be preceded by a Lagrange Multiplier. The Lagrangian Optimization of each of the eight SIAP Attributes will be mathematically

derived by obtaining the partial derivative of the attributes with respect to each of the four constraints and equating the expression to zero to enable a solution for the Lagrange Multipliers correlated to the constraints. In this mathematical formalism, the SIAP Attribute parameter objective function will be optimized for the constraints. The four constraints for each SIAP Attribute objective function parameter appearing in the following equations 3-10 are, ASoS Classification, ASoS Discrimination, ASoS Architecture and ASoS Data Registration.

**Completeness** occurs for the air picture when all objects are detected, tracked and reported. The objective function is the instantaneous system completeness at participant m at the k<sup>th</sup> scoring time t<sub>k</sub>[p.21,eqn. 1] [20].

$$C(m(t_k))_{OPT} = [C(m(t_k))] + \alpha_m [C_{1m} - ASoSA_m] + \beta_m [C_{2m} - ASoS D_m] + \gamma_m [C_{3m} ASoS C_m] + [C_{4m} - ASoS D R_{4m}] \quad (8)$$

$\alpha_m$  = Lagrange Multiplier for Architecture Constraint C<sub>1m</sub> with value ASoS A<sub>m</sub>.

$\beta_m$  = Lagrange Multiplier for Discrimination Constraint C<sub>2m</sub> with value ASoS D<sub>m</sub>.

$\gamma_m$  = Lagrange Multiplier for Classification Constraint C<sub>3m</sub> with value ASoS C<sub>m</sub>.

$\lambda_m$  = Lagrange Multiplier for Data Registration Constraint C<sub>4m</sub> with value ASoS D R<sub>m</sub>.

**Clarity** defines a clear air picture without ambiguous or spurious tracks. Tracks are ambiguous when more than one track assigned to the same object and not correlated within a system are displayed to a participant. The instantaneous system track picture ambiguity at participant m at time t<sub>k</sub> the one Clarity objective Function, (p. 23, equation 4) [20].

$$A_m(t_k)_{opt} = [A_m(t_k) + \alpha_{CA} [C_{1CA} ASoS A_{CA}] + \beta_{CA} [C_{2CA} - ASoS D_{CA}] + \gamma_{CA} [C_{3CA} - ASoS C_{CA}] + \lambda_{CA} [C_{4CA} - ASoS D R_{CA}]] \quad (9)$$

$\alpha_{CA}$  = Lagrange Multiplier for Architecture Constraint C<sub>1CA</sub> with value ASoS A<sub>CA</sub>.  $\beta_{CA}$  = Lagrange Multiplier for Discrimination Constraint C<sub>2CA</sub> with value ASoS D<sub>CA</sub>.  $\gamma_{CA}$  = Lagrange Multiplier for Classification Constraint C<sub>3CA</sub> with

value ASoS C<sub>CA</sub>.  $\lambda_{CA}$  = Lagrange Multiplier for Data

Registration Constraint C<sub>4CA</sub> with value ASoS D R<sub>CA</sub>.

**Spurious Track** is a track not assigned to any object. The Clarity ST OPT Objective Function is the instantaneous system measure of the percentage of tracks that are spurious and measured by participant m at time t<sub>k</sub> [p. 24, equation 7][20].

$$S_m(t_k)_{OPT} = [S_m(t_k) + \alpha_s [C_{1s} - ASoS A_s] + \beta_s [C_{2s} -$$

$$ASoSD_s] + \gamma_s [C_{3s} - ASoSC_{2s}] + \lambda_s [C_{4s} - ASoSDR_s] \quad (10)$$

$\alpha_s$  = Lagrange Multiplier for Architecture Constraint  $C_{1s}$  with value  $ASoSA_s$ .  $\beta_s$  = Lagrange Multiplier for Discrimination Constraint  $C_{2s}$  with value  $ASoSD_s$ .  $\gamma_s$  = Lagrange Multiplier for Classification Constraint  $C_{3s}$  with value  $ASoSC_s$

$\lambda_s$  = Lagrange Multiplier for Data Registration Constraint  $C_{4s}$  with value  $ASoSDR_s$

**Continuity** defines an air picture that is continuous when the track number assigned to an object does not change. The air picture at a point in time is represented by a collection of track reports. Continuity is the measure of the persistence and consistency of these reports over time. The Continuity Optimization Objective Function is the Rate of Track Number Changes on object  $j$  from the perspective of participant  $m$ , [p.25, equation 10] [20].

$$R_{j,mOPT} = [[R_{j,m}] + \alpha_{CO} [C_{1D} - ASoSA_o] + \beta_{CD} [C_{2D} -$$

$ASoSDR_o] + \gamma_{CR} [C_{3O} - ASoSC_o] + \lambda_{CO} [C_{4O} - ASoSDR_o] \quad (11)$   $\alpha_{CO}$  = Lagrange Multiplier for Architecture Constraint  $C_{10}$  with value  $ASoSA_o$ .  $\beta_{CO}$  = Lagrange Multiplier for Discrimination Constraint  $C_{20}$  with value  $ASoSD_o$ .  $\gamma_{CR}$  = Lagrange Multiplier for Classification Constraint  $C_{30}$  with value  $ASoSC_o$ .  $\lambda_{CO}$  = Lagrange Multiplier for Data Registration Constraint  $C_{40}$  with value  $ASoSDR_o$

**ID Completeness** occurs when all tracked objects are in an

identified state, i.e. a state other than "not at participant  $m$  at

identified." The instantaneous system ID Completeness is the percentage of objects of allegiance type  $X$  time  $t_k$  that are complete [p. 34, equation 23] [20].

$$C_{IDxm}(t_k)_{OPT} = [[C_{IDym}(t_k)] + \alpha_{CR} [(C_{1CR} - ASoSD_{CR})] + \beta_{CR} [C_{2CR} - ASoSD_{CR}] + \gamma_{CR} [C_{3CR} - ASoSC_{CR}] + \lambda_{CR} [C_{4CR} - ASoSD_{CR}]] \quad (12)$$

$\alpha_{CR}$  = Lagrange Multiplier for Architecture Constraint  $C_{1CR}$  with value  $ASoSA_{CR}$ .  $\beta_{CR}$  = Lagrange Multiplier for Discrimination Constraint  $C_{2CR}$  with value  $ASoSD_{CR}$ .  $\gamma_{CR}$  = Lagrange Multiplier for Classification Constraint  $C_{3CR}$  with value  $ASoSC_{CR}$ .  $\lambda_{CR}$  = Lagrange Multiplier for Data Registration Constraint  $C_{4CR}$  with value  $ASoSDR_{CR}$

**ID Correctness** occurs when all tracked objects are in the correct ID state. The instantaneous system ID correctness can be represented by the percentage of objects of allegiance type  $X$  labeled correctly at participant  $m$  at time  $t_k$  [p. 35, equation 27][20].

$$ID_{CXM}(t_k)_{opt} = [[ID_{CXM}(t_k)] + \alpha_R [C_{1R} - ASoSD_R] + \beta_R [C_{2R} - ASoSD_R] + \gamma_R [C_{3R} - ASoSC_R] + \lambda_{CR} [C_{4R} - ASoSDR_R]]$$

(13)  $\alpha_{CR}$  = Lagrange Multiplier for Architecture Constraint  $C_{1CR}$  with value  $ASoSA_R$ .

$\beta_{CR}$  = Lagrange Multiplier for Discrimination Constraint  $C_{2R}$  with value  $ASoSDR$ .  $\gamma_{CR}$  = Lagrange Multiplier for Classification Constraint  $C_{3R}$  with value  $ASoSDCR$ .  $\lambda_{CR}$  = Lagrange Multiplier for Data Registration Constraint  $C_{4CR}$  with value  $ASoSDR$ .

**ID Clarity** is defined clear if no tracked object is in the ambiguous ID state. The instantaneous system ambiguity  $ID_{AXM}(t_k)$  of the CID for objectives of allegiance type X at participant m at time  $t_k$  is given by, the objective function for [p. 36, equation 31] [20]:

$$ID(Axmt_k)_{opt} = [[ID_{AXM}(t_k) + \alpha_{XM}[C_{1X}ASoSA_x] + \beta_{xm}[C_{2X} - ASoSD_x] + [C_{3X} - ASoSC_x] + \lambda_{XM}[C_{4X}ASoSDR_x]] \quad (14)$$

$\alpha_{XM}$  = Lagrange Multiplier for Architecture Constraint

$C_{1X}$  with value  $ASoSA_x$ ,  $\beta_{xm}$  = Lagrange Multiplier for

Discrimination Constraint  $C_{2X}$  with value  $ASoSD_x$ ,  $\gamma_{xm}$  = Lagrange Multiplier for Classification Constraint  $C_{3X}$  with value  $ASoSC_x$  = Lagrange Multiplier for Data Registration Constraint  $C_{4X}$  with value  $ASoSDR$ .

**Commonality** defines the air picture when the assigned tracks held by each participant have the same track number position and ID. The instantaneous commonality is the Objective Function for [p.38, eqn. 35] [20]:

$$CM(t_k)_{OPT} = [[CM(t_k) + \alpha_{cm}[C_{1cm} - ASoSA_m.] + B_{cm} [C_{2cm} - ASoSD_m.] + \gamma_{cm} [C_{3cm} - ASoS C_m] + \lambda_{cm}[C_{4cm} - ASoSDR_m]] \quad (15)$$

$\alpha_{cm}$  = Lagrange Multiplier for Architecture Constraint  $C_{1cm}$

with value  $ASoSA_m$ .

$\beta_{cm}$  = Lagrange Multiplier for Discrimination Constraint  $C_{2cm}$  with value  $ASoSD_m$ ,  $\gamma_{cm}$  = Lagrange Multiplier for Classification Constraint  $C_{3cm}$  with value  $ASoSC_m$ .

$\lambda_{cm}$  = Lagrange Multiplier for Data Registration Constraint  $C_{4cm}$  with value  $ASoSDR_m$

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