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Synchronization of Chaotic Fractional-Order LU-LU System with Different Orders via Active Sliding Mode Control

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Abstract—**In this paper the main objective of this study is to investigate on chaotic behavior of fractional-order modeled LU system and its controllability. It has been shown that this problem could lead to synchronization of two master and slave systems with the different fractional-order. The proposed method which is based on active sliding mode control (ASMC) has been developed to synchronize two chaotic systems with the partially different attractor. The numerical simulation results, verify the significance of the proposed controller even for chaotic synchronization task.**

Keywords- fractional calculus; fractional order active sliding mode controller; synchronization ; LU-LU

I. INTRODUCTION

It recent years, numerous studies and applications of fractional – order systems in many areas of science and engineering have been presented [1, 2]. This is a result of better understanding of the potential of fractional calculus revealed by problems such as viscoelasticity and damping, chaos, diffusion, wave propagation, percolation and irreversibility. Fractional calculus is a more than 300 years old topic. It has useful application in many fields of science like engineering, physics, mathematical biology, psychological and life sciences [3]. In physical chemistry, the current is proportional to the fractional derivative of the voltage when the fractional interface is put between a metal and an in ionic medium [4].In the fractional capacitor theory, if one of the capacitor electrodes has a rough surface, the current passing through it is proportional to the non-integer derivative of its voltage [5]. Also the existing memory in dielectrics used in capacitors is justified by fractional derivative based models [6]. The electrode-electrotype interface is a sample of fractional-order processes because at metal-electrolyte interfaces the impedance is proportional to the non-integer order of frequency for small angular frequencies [7]. Chaotic phenomena have been observed in many areas of science and engineering such as mechanics, electronics, physics, medicine, ecology, biology, and economy. To avoid troubles arising from unusual behaviors of a chaotic system, chaos control has received a great deal of interest among scientists from various research fields in the past few decades [8]. In the recent years, emergence of effective methods in the differentiation and integration or non integer order equations makes fractional-order systems more and more attractive for the systems control community. It is verified that the fractional-order controllers can have better disturbance rejection ratio and less sensitivity to plant parameter variations compared to the traditional controllers [9]. Synchronization in chaotic dynamic systems has attracted increasing attention of scientists from various research fields for its advantages in practical application[10].A wide variety of methods have been proposed for synchronization of chaotic systems, including linear feedback control [11], sliding mode control [12],adaptive control [13] and so on. Most of the methods mentioned above are used to guarantee the asymptotic stability of chaotic systems. Among the fractional order controllers, the fractional order active sliding mode control h (FOASMC) has been dealt more than others. In this paper, we introduce a fractional-order systems chaotic LU. To control and synchronization of chaotic fractional-order system an active sliding mode controller (ASMC) is proposed. This novel control law makes the system states asymptotically stable, simulation result show that the presented control method can easily eliminate chaos and stabilize the market. The rest of the paper is organized as follows.

II. FRACTIONAL-ORDER DERIVATIVE AND ITS APPROXIMATION

A. Definition

The differ integral operator, represented by $0^{D_t^{\alpha}}$, is a combined differentiation-integration operator commonly

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used in fractional calculus and general calculus operator, including fractional-order and integer is defined as:

$$0^{D_t^{\alpha}} = \begin{cases} \frac{d^{\alpha}}{dt} & \alpha > 0 \\ 1 & \alpha = 0 \\ \int_0^t (d\tau)^{-\alpha} & \alpha < 0 \end{cases} \quad (1)$$

There are several definitions of fractional derivatives [14]. The best-known one is the Riemann-*Liouvile* definition, which is given by

$$\frac{d^{\alpha}f(t)}{dt^{\alpha}} = \frac{1}{\Gamma}\frac{d^n}{dt^n}\int_o^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}}d\tau \quad (2)$$

Where n is an integer such that$n-1<\alpha<n$,$\Gamma(0)$ is the Gamma function. The geometric and physical interpretation of the fractional derivatives was given asfollows

$$\Gamma(z) = \int_0^{\infty} e^{-t}t^{z-1}dt \quad (3)$$

The Laplace transform of the Riemann- *Liouville* fractional derivative is

$$L\left\{\frac{d^{\alpha}f(t)}{dt^{\alpha}}\right\} = s^{\alpha}L\{f(t)\} - \sum_{k=o}^{n-1} s^k [\frac{d^{\alpha-1-k}f(t)}{dt^{\alpha-1-k}}] \quad (4)$$

Where, L means Laplace transform, and s is a complex variable. Upon considering the initial conditions to zero, this formula reduces to

$$L\left\{\frac{d^{\alpha}f(t)}{dt^{\alpha}}\right\} = s^{\alpha}L\{f(t)\} \quad (5)$$

The Caputo fractional derivative of order α of a continuous function $f : R^+ \to R$ is defined as follows

$$\frac{d^{\alpha}f(t)}{dt^{\alpha}} = \begin{cases} \frac{1}{\Gamma(m-\alpha)}\int_0^t \frac{f^{(m)}(\tau)}{(t-\tau)^{\alpha-m+1}} & m-1<\alpha<m \\ \frac{d^m}{dt^m}f(t) & \alpha = m \end{cases} \quad (6)$$

Thus, the fractional integral operator of order α can be represented by the transfer function $H(s) = \frac{1}{s^{\alpha}}$ in the frequency domain.

The standard definition of fractional-order calculus does not allow direct implementation of the fractional operators in time-domain simulations. An efficient method to circumvent this problem is to approximate fractional operators by using standard integer-order operators. In Ref.[15], an effective algorithm is developed to approximate fractional-order transfer functions, which has been adopted in [16] and has sufficient accuracy for time-domain implementations. In Table 1 of Ref [17], approximations for $1/_{s^{\alpha}}$ with α from 0.1 to 0.9 in step 0.1 were given with errors of approximately 2 dB. We will use the $1/_{s^{0.95}}$ approximation formula [16] in the following simulation examples.

$$\frac{1}{s^{.95}} \approx \frac{1.2831s^2 + 18.6004s + 2.0833}{1.2831s^3 + 18.4738s^2 + 2.6574s + 0.003} \quad (7)$$

In the simulation of this paper, we use approximation method to solve the fractional-order differential equations.

III. DESIGNING THE FRACTIONAL-ORRDER ACTIVE SLIDING MODE CONTROL AND ANALYSIS

To design the active sliding mode controller, we have procedure a combination of the active controller and the sliding mode controller.

A. Active sliding mode controller design

Let us, consider a chaotic fractional-order description of the system as follows

$$0^{D_t^{\alpha_1}}X_1 = A_1X_1 + g_1(X_1) \quad 0<\alpha<1 \quad (8)$$

Where $X_1(t) = (x_1, x_2, x_3)^T$ are real state vector, $A_1 \in R^{3\times3}$ denotes the linear part of the system dynamics and $g_1: R^3 \to R^3$ is nonlinear part of the system. Eq. (1) denotes the master system. Let $X_0 = (x_{10}, x_{20}, x_{20})^T$ be the any initial conditions in the chaos attractor of fractional-orders system (8).

Now the controller $u(t) \in R^3$ is added the slave system.

Thus:

$$0^{D_t^{\alpha_2}}X_2 = A_2X_2 + g_2(X_2) + u(t) \quad 0<\alpha<1 \quad (9)$$

That X_2, A_2 and g_2 implies the same roles as X_1, A_1 *and* g_1 for the master system.Synchronization of the systems means finding a control signal $u(t) \in R^3$ that makes state of the slave system to evolve as the states of the master system.

Now we define errors dynamics as follows

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$$0^{D_t^{\alpha_2}} X_2 - 0^{D_t^{\alpha_1}} X_1 = A_2X_2 + g_2(X_2) - A_1X_1 - g_1(X_1) + u(t) \quad (10)$$

Now following sentence add to the equation (10)

$$0^{D_t^{\alpha_2}} X_2 - 0^{D_t^{\alpha_2}} X_2$$

Thus:

$$0^{D_t^{\alpha_2}} X_2 - 0^{D_t^{\alpha_1}} X_1 = A_2X_2 + g_2(X_2) - A_1X_1 - g_1(X_1) + u(t) - 0^{D_t^{\alpha_2}} X_2 + 0^{D_t^{\alpha_1}} X_1 \quad (11)$$

That: $e = X_2 - X_1$ *and* $A_1 = A_2 = A$

Thus:

$$0^{D_t^{\alpha_2}} e = A_2X_2 + g_2(X_2) - A_1X_1 - g_1(X_1) + u(t) - 0^{D_t^{\alpha_2}} X_2 + 0^{D_t^{\alpha_1}} X_1 \quad (12)$$

Now we assump:

$$G(X_1, X_2) = g_2(x_2) - g_1(x_1) + (A_2 - A_1)x_1 - 0^{D_t^{\alpha_2}} X_2 + 0^{D_t^{\alpha_1}} X_1 \quad (13)$$

The aim is to design the controller $u(t) \in R^3$ such that:

$$\lim_{t\to\infty} \| e(t) \| = 0 \quad (14)$$

Then use with the active control design procedure [19, 20]

U (t) change as following:

$$u(t) = H(t) - G(X_1, X_2) \quad (15)$$

Eq. (15) describes the newly defined control input H (t). Where *H(t)* is:

$$H(t) = Kw(t) \quad (16)$$

Where $k \in R^3$is a constant gain vector and $w(t) \in R$ is the control input that satisfies in:

$$W(t) = \begin{cases} w^+(t) & s(e) \geq 0 \\ w^-(t) & s(e) < 0 \end{cases} \quad (17)$$

Where $s = s(e)$ is a switching surface that describes the desired dynamics the resultant error is then written by

$$0^{D_t^{\alpha_2}} e = Ae + K\,W(t) \quad (18)$$

B. Constructing a sliding surface

Constructing a sliding surface which represents a desired system dynamics and the sliding surface described as follows

$$s(e) = Ce \quad (19)$$

Where $C \in R^3$is a constant vector. An equivalent control is found when $\dot{S}(e) = 0$which is a*necessary condition* for the state trajectory to stay on the switching surface $S(e) = 0$ hence, the controlled systemsatisfies the following conditions in the steady state:

$$S(e) = 0 \quad and \quad \dot{S}(e) = 0 \quad (20)$$

Based on equation (18) to (20), it could be deduced:

$$\dot{S}(e) = \left(C0^{D_t^{1-q_2}} (Ae + kw(t)\right) = 0 \quad (21)$$

Thus,

$$0^{D_t^{1-\alpha_2}} w(t) = -(ck)^{-1} CA \left(0^{D_t^{1-\alpha_2}} e(t) \right) \quad (22)$$

A solution of Eq.22 is

$$w_{eq}(t) = -(CK)^{-1} CAe(t) \quad (23)$$

C. Sliding mode control of fractional order system

We consider the constant plus proportional rate reaching Law will be considered [18]. Accordingly the reaching law is obtained as:

$$0^{D_t^{\alpha_2}} S = -\rho \operatorname{sgn}(s) - rs \quad (24)$$

That $sgn(0)$ represents the sign function. They ρ , r are gains that the sliding conditions Eq. (20) are satisfied. From Eqs (18), (19) have:

$$0^{D_t^{\alpha_2}} S = C0^{D_t^{\alpha_2}} e = C[Ae + kw(t)] \quad (25)$$

From Eqs (23) and (24) find control effort can be defined as:

$$w(t) = -(CK)^{-1}[C(rI + A)e + \rho \operatorname{sgn}(s)]. \quad (26)$$

D. Stability

First, we represent stability theorems from the fractional calculus.

Theorem 1($Matignon$ [19]). The following system:

$$0^{D_t^{\alpha}} = Ax, \quad x(0) = x_0 \quad (27)$$

Where $0 < \alpha < 1, x \in R^n$ and $A \in R^{n\times n}$, is asymptotically stable if $|\arg(eig)\, A| > \alpha\pi/2$.

According to Theorem 1, as long as all eigenvalues of $[A - K(CK)C(rI + A)]$ $(\lambda_i = 1,2,3)$ satisfy the conditions $|\arg(\lambda_i)| > \alpha\pi/2$, the system is asymptotically stable.

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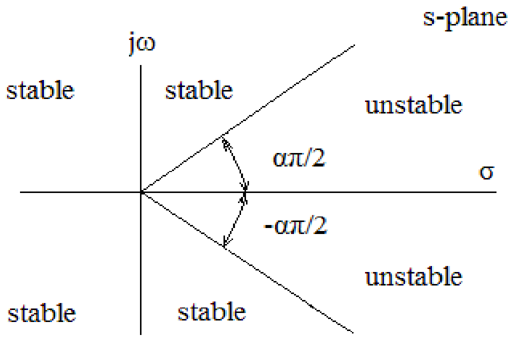

Figure 1. Stability region of linear fractional-order system wih α

IV. Numerical Simulations

A. *Synchronization between two fractional-order lu systems*

The LU system [20], was introduced by Chen and Lu
.

$$\begin{cases} {}_0D_t^{\alpha}x = \rho(y-x) \\ {}_0D_t^{\alpha}y = -xz+\nu y. \\ {}_0D_t^{\alpha}z = xy-\mu z \end{cases} \tag{28}$$

For this system matrix A is

$$A = \begin{bmatrix} -\rho & \rho & 0 \\ 0 & \nu & 0 \\ 0 & 0 & -\mu \end{bmatrix} \tag{29}$$

In this section, we consider using (ASMC) technique to obtain synchronization. This controller guarantees the synchronization two fractional orders Lu systems with the following initial conditions:

$$(x_{10}, y_{10}, z_{10}) = (1,4,10)$$
and
$$(x_{20}, y_{20}, z_{20}) = (0,3,9).$$

Consider two fractional order LU systems as master and slave systems respectively:

$$\text{Master system} \begin{cases} {}_0D_t^{\alpha_1}x_1 = 35(y_1-x_1) \\ {}_0D_t^{\alpha_1}y_1 = -x_1z_1+28y_1. \\ {}_0D_t^{\alpha_1}z_1 = x_1y_1-3z_1 \end{cases} \tag{30}$$

$$\text{Slave system} \begin{cases} {}_0D_t^{\alpha_2}x_1 = 35(y_2-x_2) \\ {}_0D_t^{\alpha_2}y_2 = -x_2z_2+28y_2. \\ {}_0D_t^{\alpha_2}z_2 = x_2y_2-3z_2 \end{cases} \tag{31}$$

Assume that order of the master is $\alpha_1 = 0.88$ and order the slave is $\alpha_{2=}0.9$. Parameters of the controller are chosen as $k = [-3,-6,-1]^T$, $C = [1,1,-1]$, $r = 85$ and $\rho = 0.35$. This selection of parameters results in Eigen values $[\lambda_1, \lambda_2, \lambda_3] = [-85, -37.9523, -6.5477]$which Located in a stable region($|\arg(\lambda_i)|) > {}^{\alpha\pi}/_2$. Fig. shows the effectiveness of the proposed controller to synchronize two fractional-order modeled systems. It should be noted that control u (t), has been activated at $t = 0$. the simulation results are shown in Fig.2.
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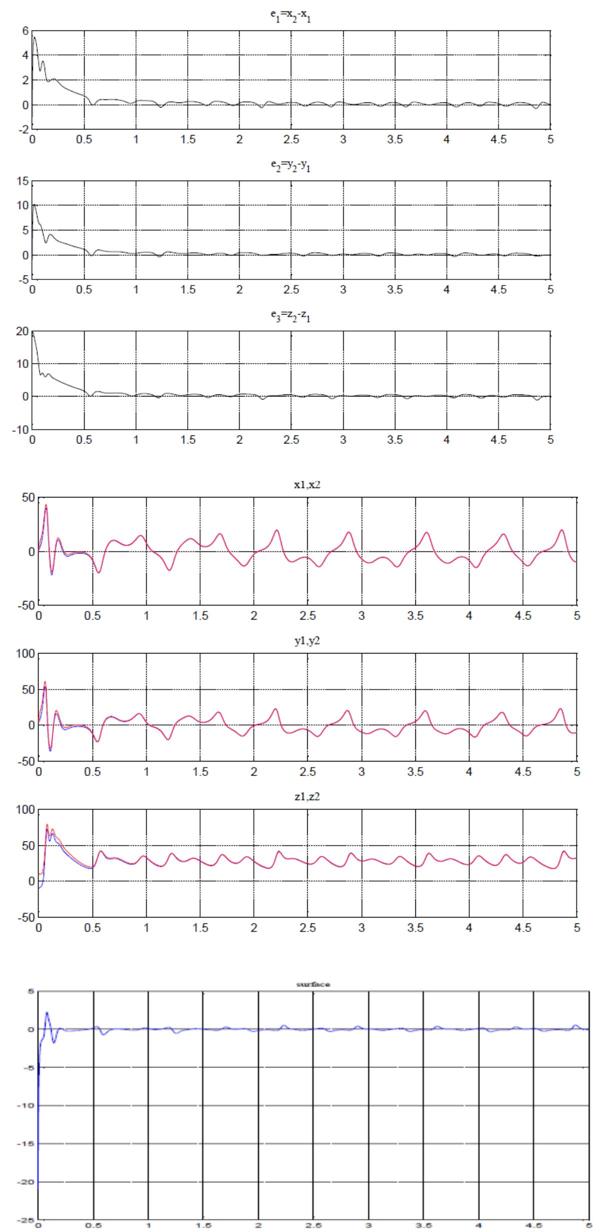

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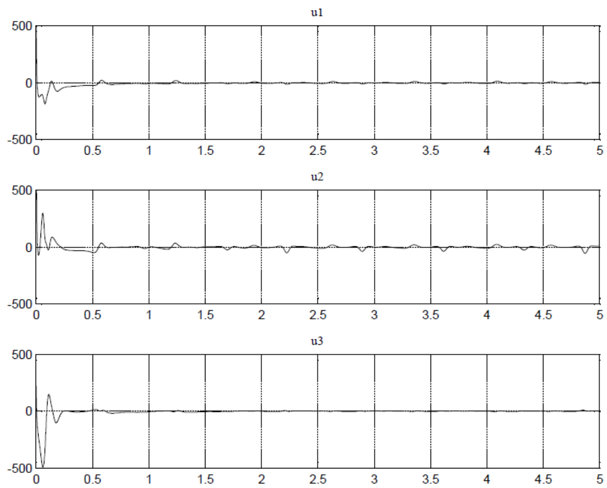

Figure 2. Results of simulaton.

V. CONCLUATION

This paper we have studied numerical methods in fractional calculus. Then, we have represented the active sliding mode control to synchronize. The control parameters $(r, k\ and\ c)$, the master and slave systems are synchronized. Numerical simulations show the efficiency of the proposed controller to synchronize chaotic fractional-order.

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