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Systems Engineering: Optimization on Stiefel Manifold for MIMO System

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Abstract— **Applications of multiple input and multiple output (MIMO) with Stiefel manifold are growing in the next generation communications and their system engineering developments. These applications need to be optimized efficiently with less complexity and cost. In this research, optimization problems in MIMO systems using Stiefel manifold are considered. As far as general manifolds are concerned, optimization methods are applied in a different environment that deals with basic signal processing. Here, specific optimization technique, which can be applicable in current MIMO system with Stiefel manifold, is given as a methodology. The current MIMO system is a new technology, and it can be applied in the next generation technology because it uses manifolds in feedback of MIMO systems. Even though, overall performance of new and current technology is better than conventional techniques, final performance of MIMO applications is anticipated with efficient optimizations.**

Keywords-Stiefel manifold; MIMO system; optimization; Feedback.

I. Introduction

In multiple-input and multiple-output (MIMO) system, optimization is inevitable because it is employed in many application of communication system engineering. In order to minimize the error between the small balls within the manifolds, optimization that provides best results with less iteration is very important in MIMO channel modeling. The speed of the wireless system and communication is still challengeable problem because of fading and spectrum scarcity in MIMO channels. In the next generation MIMO system, optimizing parameters related to system engineering or communication system engineering will be key areas or challenges to increase the data rate. The metric learning approach for increasing the rate in MIMO is based on the assumption that the two optimization problems, i.e. maximizing classification performance and minimizing volume elements are indeed correlated.

Manifolds and their optimization techniques are studied to find the optimum point of manifold, which is the dimension of large matrices and/or data set used in MIMO applications. Stiefel manifolds are dominating many fields in wireless communication successfully. This type of manifold is useful for limited and finite rate feedback, precoding, channel estimation and modulation. Signal processing tools for Stiefel manifold have been developed to optimize the wireless systems. In this optimization, time series on the Stiefel manifold should be analyzed to develop the efficient algorithms. Further, signal processing of MIMO depends on the filters, equalizers and other components, which need to be optimized. Interpolating manifolds are also considered minimizing the distortions between the points on the manifold.

In 1935, Stiefel introduced the manifolds analyzing from basic to higher order differentiable manifolds with orthonormal vector$a_1, a_2, a_3, \dots, a_i \in \mathbb{F}^p$, where Euclidean space is represented as p-dimensional ($\mathbb{F}^p$) and $k \leq p$ [12]. Optimization problems, which tend to affect in receiving signals because channel matrix in MIMO is involved with subspaces. In Stiefel manifold, constraint $Q^T Q = I$, require column matrices of Q to have orthogonal property. It means that the cost function f(Q) is involved with a set of orthonormal vectors.

We consider a single user communication and a point-to-point link where the transmitter is equipped with N_t antennas and receiver with N_r antennas [1]. Overhead problem is increased with square of total antennas used in the transmitter. Distance between the two points on planes of the n-dimension object. The geodesic and chordal distances should be defined clearly and optimized with dimensions of the manifold. A geodesic, which is a curve given between any two points located on the surface of any manifolds [11], can be assumed as a straight line in Euclidean space because in the context of distance properties, two points are very closed each other.

Singular value decomposition (SVD) of the matrix obtained from channel matrix and eigenvectors associated to this matrix and other related calculations in MIMO need to

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be improved through the appropriate optimization techniques [10]. Linear transceiver based on SVD with optimization can be considered as linear detection in an open-loop MIMO system.

The sections and sub-sections in the paper are categorized as follows. Section II provides basic information of system engineering and detailed background of MIMO communication system engineering. Section III introduces the optimization techniques used in latest and future MIMO system. In section IV, results and analysis of optimization on Stiefel manifold are tabulated. Overall conclusions are summarized in Section V.

II. System Engineering

The MIMO system is part of communication system engineering. The MIMO system is engineered with many sub-systems such as filtering and equalizations which are a part of the MIMO receiver. Computational complexity in MIMO and convergence speed of channel matrices during the communication are parts of the optimization problems.

A. Basic MIMO system

As a part of the communication system engineering, basic MIMO system can be considered. Here, system model known as (1) should be optimized. In this, a number of transmitting and receiving antennas needs to be analyzed because antennas quality and quantity affecting channel matrices of MIMO system. MIMO channel is complicated when number of antennas is increased in some applications. Channel matrix, which is basically a number of antennas, needs to be optimized when it is involved with receiver implementation of MIMO.

$$Y_i = \sqrt{\rho} X_i H_i + \eta_i \qquad (1)$$

Here, X_i , Y and η_i are transmitted, received and noise vectors, such as

$$X_i = \{X_1, X_2 ... X_{Nt}\}$$
$$Y_i = \{Y_1, Y_2 ... Y_{Nr}\}$$
$$\eta_i = \{\eta_1, \eta_2 ... \eta_{Nr}\}$$

H is the channel matrix $N_t \times N_r$ matrix denoting MIMO channel influenced with transmit and receive antennas. Here, the constant ρ is the signal to noise ratio at each receiver antenna. In MIMO, power optimization of signal, noise and interference signal is also needed because power of interference signals affects the signal conditions [3].

The improvement of channel capacity is considered when MIMO systems are employed without using extra bandwidth or power. The capacity of the MIMO channel can be obtained by using different algorithms [9] such as equal power allocation, beamforming and waterfilling respectively. Regarding the MIMO context, this algorithm is also to be optimized using efficient optimization algorithms. In digital signal processing, beamforming is already optimized for different application. Optimization of Decoding in MIMO is also one of the key problems in system engineering.

B. The MIMO system with feedback

Latest applications use the MIMO system with feedback, which needs best optimization techniques because many challenges are depending on appropriate feedback designs. Figure 1 shows some of the necessary components used in the MIMO communication and its system engineering. In order to satisfy the capacity requirements on the feedback channel, an efficient quantization of channel state information (CSI) is mandatory. In MIMO applications, CSI controls the functions of receiver components, which are equalization, detection, etc. In order to implement perfect or limited CSI, source of the transmitter fed back from receiver needs to be optimized. Linear precoding based on CSI is another component, which needs compatible optimization techniques.

In order to optimize the feedback, we understand the quantized feedback model and quantization codebook design that can be used for the precoding [2]. Quantization is influenced with Stiefel manifold and distances between any two points on the manifold. Each point is assumed as matrix based on MIMO channel matrix. Following equation can be used for performance affection with quantized feedback [8].

$$D(w) = E_H \left[\lambda_1 - \max_{\omega \in w} \sum_i \lambda_i \, | v_i^H \omega |^2 \right] \qquad (2)$$

.

Here, D(w) is the distortion measurement, which affects the BER performance. In equation (2), w is the projection codebook and value λ_1 is the largest value of the $H^H H$.

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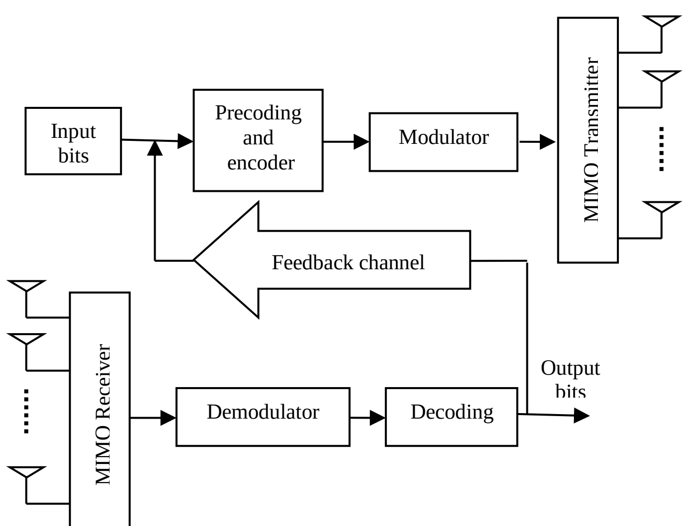

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Figure 1. MIMO system with feedback

C. The MIMO system with manifolds

In quantized feedback scheme, quantizing the channel or quantizing the properties of the received signal considered as the main approach is applied with manifolds. For MIMO system, the sphere packing bounds and distortion rate tradeoff are addressed as the basic quantization problems. The quantization of precoding (beamforming) matrices is influenced to some manifolds in which MIMO system with finite rate feedback is considered. In order to understand the specific and various quantization problems on Stiefel manifold, the quantization on the Grassmann manifold given in [2][4] is analyzed to explain the following points.

The complex Stiefel manifold $V_{p,k}^{\mathbb{C}}$ is the set of p-tuples of orthonormal vectors (3).

$$V_{p,k}^{\mathrm{C}} = \{Q \in \mathrm{C}^{k\times p} \mid Q^T Q = I_k\} \quad (3)$$

Where I_P is the $P \times P$ identity matrix. If Stiefel manifold has matrix Q with full column rank, unique solution will be expected. According to [9][12], Stiefel manifold is considered as key matrices, which enhances the receiver performance of the basic MIMO system. The space of orthonormal matrices, which is rectangular with k < p have associated in definition 1. The C used in equation (3) is representing the complex field of Stiefel manifold.

Definition 1: The definition of complex Stiefel manifold is defined as

$$Y \in V_{p,k}^{C} \quad | \quad \sum_j |Y_{i,j}|^2 = k/p \quad \forall i \quad (4)$$

In equation (4), k/p is a constant value on the main diagonal of the matrix, which is a point on the manifold.

Definition 2: The complex dimension of the Stiefel manifold can be defined that it is the sum of dimension of skew-Hermitian matrices and the dimension of $n \times (n-k)$ matrices.

$$\dim V_{p,k}^{\mathrm{C}} = 0.5k(k-1) + k(p-k) \quad (5)$$

In conventional techniques, MIMO channel is quantized directly using the covariance matrix of the MIMO channel. Here, quantization scheme is developed from the matrices allocated in the Stiefel manifold. So, codewords are generated from the predefined codebook, which contains Stiefel properties.

The quantization on Stiefel manifold can be considered with the following theorem because the dimension of Stiefel manifold can be characterized through this theorem.

Theorem 3 [16] The set $V_{p,k}^{\mathbb{C}}$ is a compact differentiable with dimension $kp - k(k+1)/2$ for every pair of positive integers (k, p) satisfying $k \le p$. Stiefel manifolds are connected if k<p. When k = p, the "Stiefel manifolds" have two components.

D. Optimization on complex Stiefel manifold

The main principle behind optimization on manifolds is to rewrite the optimization problem in terms of a local parameterization at each iteration. A local parameterization about the point

III. TECHNIQUES OF OPTIMIZATION IN MIMO SYSTEM

The MIMO system is optimized by many techniques, which reduce the cost of the system. New MIMO systems are influenced with manifolds, which increase the rate through the quantization.

The two types of algorithm used in this paper are based on the traditional optimization algorithm [14]. Although [13][15] derived similar type algorithms to the ones presented here, there is an important difference; in [13], the pertinent manifold was locally parameterized by using the exponential map, whereas this paper locally parameterizes the manifold by a Euclidean projection of the tangent space onto the manifold. This difference affects the computational complexity and the rate of convergence of the algorithms employed in MIMO system. The performance of the optimization algorithms resulting from the two different parameterizations was compared in [15]. When Stiefel manifold is applied in particular cost function, the Euclidean projection of the tangent is calculated using optimization techniques. In this research, Newton's method provides less computational complexity and faster convergence.

A. Newton's optimization technique

This technique is given in [12] for different applications but in MIMO context, Newton's method for minimizing $F(Q)$ on the Stiefel Manifold.

First iteration **:** Given Q such that $Q^T Q = I_k$,

Compute $G = F_Q - QF_Q^T Q$

Compute $\Delta = -Hess^{-1} G$ such that $Q^T \Delta = skew - symmetric$

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$$F_{QQ}(\Delta) - Qskew(F_Q^T \Delta) - skew(\Delta F_Q^T)Q - 0.5\prod \Delta Q^T F_Q = -G$$

where $skew(X) = 0.5*(X - X^T)$ and $\prod = I_k - QQ^T$.

Move from Q in direction Δ to Q(1) using the geodesic formula

$$Q(t) = QM(t) + PN(t) \tag{6}$$

Here, PR is the compact PR decomposition $(I_k - QQ^T)\Delta$.

$P \in \mathbb{F}^{p\times k}$ and $R \in \mathbb{F}^{k\times k}$ are used in (6) and (7) respectively.

$A = Q^T\Delta$, M(t), and N(t) are k-by-k matrices given by the 2k-by-2k matrix exponential

$$\begin{pmatrix} M(t) \\ N(t) \end{pmatrix} = e^t \begin{pmatrix} A & -R^T \\ R & 0 \end{pmatrix} \begin{pmatrix} I_k \\ 0 \end{pmatrix} \tag{7}$$

Repeating iteration: above procedures are repeated continuously as shown in figure 2 until the error is optimized.

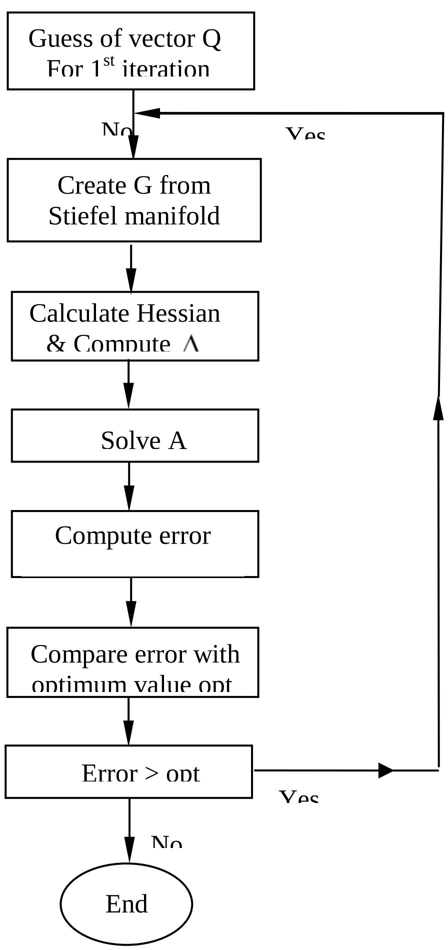

Figure 2. Newton's method for Stiefel manifold

The above flow chart shows the basic steps of Newton's method that calculates the error. With the optimum value, optimization on Stiefel manifold is computed and applied to all necessary components of MIMO system.

B. Conjugate gradient technique

As a conventional technique, conjugate gradient method is studied. Optimization with minimizing $F(Q)$ on the Stiefel Manifold is described in [15]. Here, necessary steps of this conventional technique are mentioned for the comparison.

Given Q such that $Q^T Q = I_k$,
Compute $G = F_Q - QF_Q^T Q$ and set $\alpha_0 = -G_0$

For i = 0,1,…,
Minimize $F(Q(t))$ over t where

Q(t) = QM(t) + PN(t)

PR is the compact QR decomposition of $(I - Q_i Q_i^T)\alpha_i$ $A = Q_i^T \alpha_i$, and $M(t)$, and $N(t)$ is k-by-k matrices given by the $2k$-by-$2k$ matrix exponential appearing in Newton's method on the Stiefel manifold in previous sub-section [15].

Set $t_i = t_{min}$ and $Q_{i+1} = Q_i(t_i)$
Compute $G_{i+1} = F_{Q_{i+1}} Q_{i+1} F_{Q_{i+1}}^T Q_{i+1}$
Parallel transport tangent vector α_i to that point Q_{i+1}:

$$\tau\alpha_i = \alpha_i M(t_i) - Q_i R^T N(t_i) \tag{8}$$

As discussed above, set $\tau G_i = G_i$ or 0. (It is not parallel.)
Compute the new search direction

$$\alpha_{i+1} = -G_{i+1} + \gamma_i \tau\alpha_i \tag{9}$$

where

$$\gamma_i = \frac{\langle G_{i+1} - \gamma G_i, G_{i+1} \rangle}{\langle G_i, G_i \rangle} \tag{10}$$

$$\langle \Delta_1, \Delta_2 \rangle = tr\Delta_1^T (I - 0.5 \times Q_i Q_i^T)\Delta_2 \tag{11}$$

Reset $\alpha_{i+1} = -G_{i+1}$

if $i + 1 \equiv 0 \; mod \; k(p-k) + k(k-1)/2$.

In this method, calculations need more steps and procedures than Newton's method compared in the next section.

IV. RESULTS AND ANALYSIS

In this optimization, the Stiefel manifold $V_{8,4}$ is considered as an example (p = 8 and k = 4). So using equation (5), dimensions of Stiefel manifold can be calculated. The dimension of $V_{8,4}$ equals 4(4 - 1)/2 + 16 = 22; therefore, the accuracy of the conjugate gradient should double every 22 iterations. We analyzed dimensions of $V_{8,4}$ and $V_{6,4}$ through Newton's and conjugate gradient methods. From these two examples, Newton's method minimized the cost and complexity because the error is minimized within five iterations. Variable precision numerical software is used to demonstrate the asymptotic convergence properties of these algorithms.

A. Analysis and discussion

Figure 3 shows the optimization solution for specific manifolds used in MIMO systems. In this analyses, $V_{6,4}$ and $V_{8,4}$ are considered to verify the algorithms in our research of which communication system engineering is implemented for current MIMO applications.

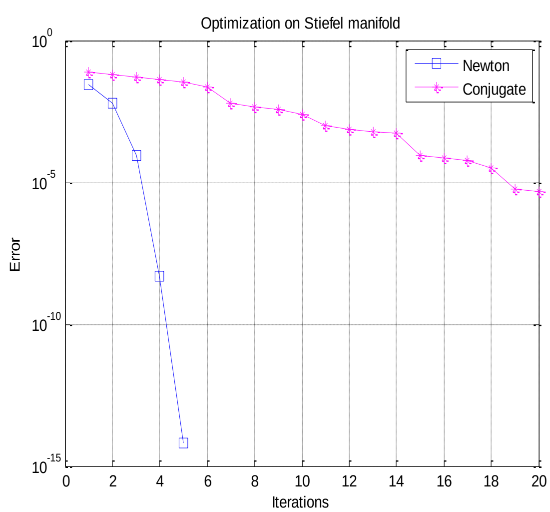

Figure 3. Comparison of Newton and Conjugate methods

B. Recommendations

Feedback data can be either sent through the Additive White Gaussian Noise (AWGN) channel or MIMO channel. In both cases, optimization will be analyzed using efficient optimization techniques because CSI can be enhanced in latest MIMO applications. Still, there are many components in MIMO communication system should be optimized. So, system engineering of this research needs to be examined with appropriate optimization techniques. Quantization is one of the recommended areas in this research. Quantization in the receiver of MIMO system where each element obtained from Stiefel manifold is quantized with different rate and tested. Computing minimum eigenvectors associated with eigenvalues is one of the advantages when Stiefel manifold is employed in quantization development.

Functions of MIMO on Stiefel manifold based on channel matrix need to be analyzed using optimality conditions derived from which first to higher order and convergent algorithms are employed in MIMO system.

We can apply geometric optimization tools for finding the nearest low-rank correlation matrix, which will help us to optimize necessary components used MIMO system.

V. CONCLUSIONS

In this research, we studied optimization techniques for a next generation approach considered on a Stiefel manifold that is popular research topic in MIMO system.

From the results, quantized feedback and the Stiefel manifold increased the performance of the overall MIMO system because error reductions in characteristics of feedback and Stiefel manifold are optimized.

Feedback channel is analyzed with different rates, which increase the minimum distance when rate is increased. In order to increase the minimum distance accurately, we introduced the Newton's method as an efficient optimization technique. This method provides quick optimization that means optimum result is achieved within six or less than six iterations as in figure 3. We studied the quantization on manifolds, where each point on the Stiefel manifold is quantized using different rate. Hence, optimization, which is one of the key points of system engineering should optimize the quantization error and increase the overall performance of MIMO system.

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