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A Knowledge Based Neuro-Fuzzy Model and Controller Synthesis of a Highly Nonlinear Dynamics Electrical Machine

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Abstract. It is important to drive robotics systems with heavy duty machines. However, analysis and controller synthesis of an electrical rotating machine is considered as hard task to be achieved. This is due to the complicated and nonlinear differntional equations that govern such types of electro-mechanical systems. This article has been conducted to solve the issue of designing linear controllers (even with some Robust H_∞ characters) for a class of nonlinear electrical motor. Initially a Takagi – Sugeno (T-S) Neuro-Fuzzy models are built while extracting machine sub-linear models. Local state feedback controllers are synthesized using some optimization tools. For designing the controller with some noise rejection characters, an H_∞ was used, while solving LYAPUNOV candidate function using LMI formulation. The synthesized controller strategy has proven as an effective in terms of solving for optimal system algebraic Riccati formulation, while relying on Neuro-Fuzzy sub models.

Keywords: *Neuro-fuzzy Modeling, Patterns Clustering , Nonlinear Dynamics, Robotics Drives*

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1. Introduction

1.1. Robotics drives, electric machine modelling and control

Nonlinear H_∞ control schemes have been initiated to deal with robust performance design requirements systems with nonlinear behaviors, [1]. For the case of conventional optimal controller design, a plant model must be known. For such a case, it is needed to evaluate the well-known Hamilton-Jacobi equation, (a class of nonlinear partial differential equation, [2]). The well-known standard nonlinear H_∞ control design, are in fact, not appropriate for practical control system design. Hence, control engineers do refer to a simple fuzzy control design with guaranteed control performance. To stabilize nonlinear systems, fuzzy controllers have introduced. This helps to eliminate effects of external disturbances below a prescribed levels. By doing so, desired H_∞ control performance is guaranteed, Chen [3]. Due to simplicity in design, merely linear fuzzy control is used, without complicated feedback linearization or parameter update law, though, H_∞ performance is achieved with minimized attenuation level. The introduced method is an attempt to combine the merits of linear fuzzy model and H_∞ performance to obtain a simple but practical algorithm. The approach is considered a bridge among two significant design techniques. This is the robust and fuzzy control paradigm. Accordingly the technique is to grant the additional H_∞ design with intelligence and fuzzy technique with better performance respectively, [4],[5], and [6]. In addition, the technique of linear matrix inequality (LMI), has also emerged as vigorous tool through solving the known Algebraic Riccati Equation. For instance, Li et. al. [6], showed the relation between LMIs and AREs through absolute stability criteria, robustness analysis and optimal control. An analysis for robust stability of a

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fuzzy control system via quadratic stabilization, H_∞ control theory can in fact be solved by LMI techniques. Neuro-fuzzy architectures have the ability to even model small inherent dynamics in electrical machines. Hence, they do bridge gaps between **AI** related modeling techniques, and controllers synthesis. Such a charterers are not easily available within other modeling methods.

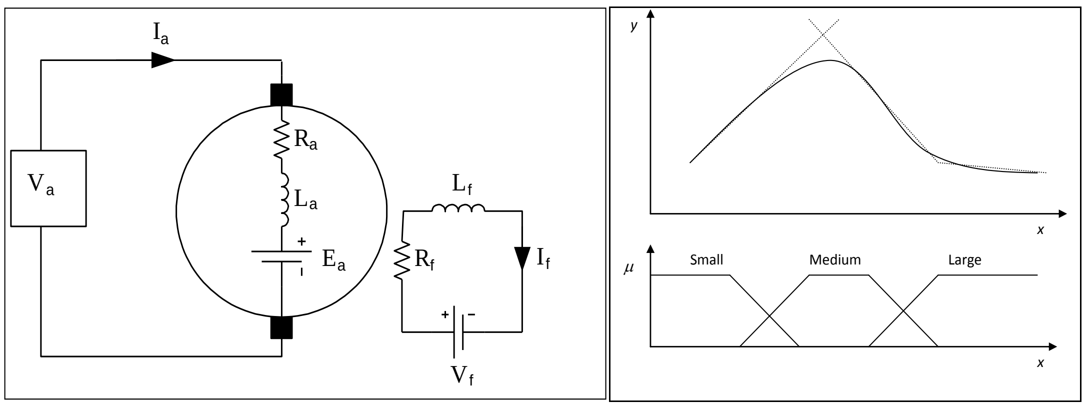

Figure 1. Bridging the gap between machine engineering and soft computing techniques.
(*a*) Machine equivalent circuit. Machine interrelation dynamics are highly nonlinear.
(*b*) Takagi –Sugeno linear fuzzy models, seen as a smoothed piece-wise linear approximation.

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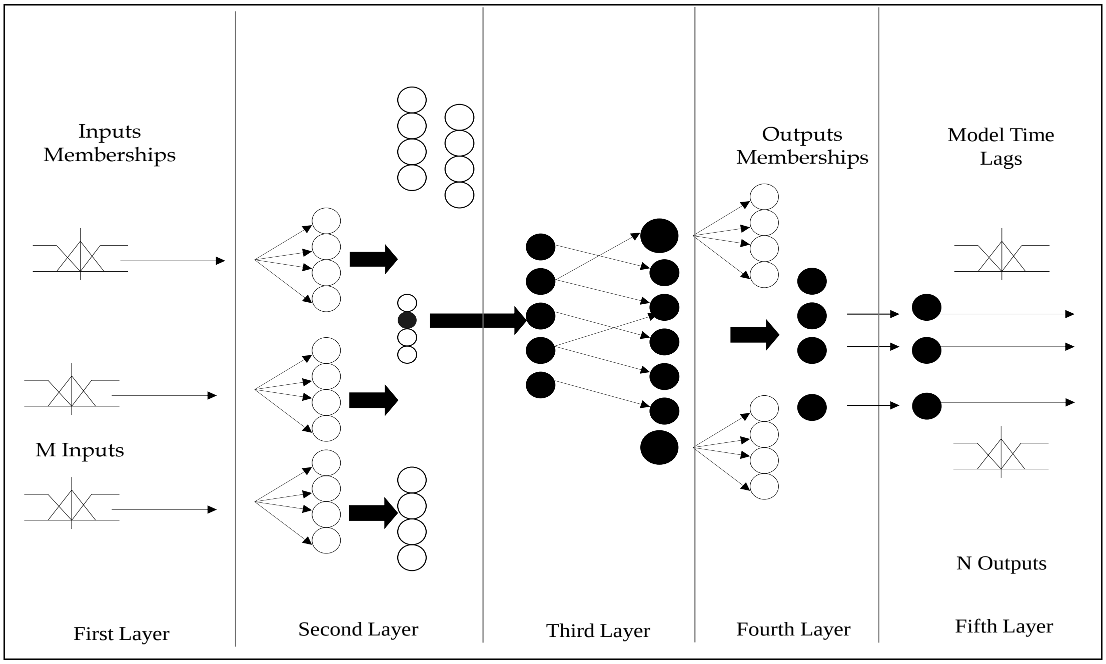

Figure 2. Learning machine dynamics with a five layers Neuro-Fuzzy architecture.

1.2. Research objective

The approach presented here is an attempt to combine linear fuzzy models and the H_∞ performance to obtain an algorithm for a robust control of a nonlinear electrical machine (ac motor). For the machine details, this is illustrated in fig. 1. The figure shows the corresponding schematic diagram of the adopted totally nonlinear electrical machine, with the associated dynamic parameters. Hence, the approach is to incorporate the obtained linear (T-S) fuzzy models (via a five layers Neuro-fuzzy architecture), to design an H_∞ characterized controller system. The used Neuro-fuzzy architecture is shown in fig. 2. The approach is bridging the gap between computationally **AI** related modeling techniques and others advanced controller synthesis. The presented nonlinear electrical machines are highly nonlinear dynamic systems due to magnetization, therefore it was made easy to model it with multiple linear models, as achieved via the adopted Neuro-fuzzy architecture.

1.3. Manuscript organization

This article has been organized into six main sections. Section (1) presents a brief introduction to the subject and related challenges. In section (2), we present machine dynamics and the basis of fuzzy (T-S) models. It also present the details of a five layers Neuro-Fuzzy architecture. Section (3) discusses H_∞ design for fuzzy linear models. Section (4) presents the problem formulation of a closed loop Lyapunov based H_∞ controller synthesis. In section (5) we experiment and show a case study for the proposed controller synthesis. Finally section (6) draws few points of conclusion.

2. Machine dynamics and model description

2.1. Nonlinear machine dynamics

The machine interrelation parameters dynamics are considered nonlinear. In reference fig. 1, the detailed system parameters are presented here. Numerical parametric details of the machine will be discussed in section (5). For notations, (i_a) is the machine armature current, (i_f) is the machine field current, (T_{load}) is machine load torque, (T_e) is the electromagnetic torque. Φ_f is the field winding flux linkages, ω_r rotor speed in rad/s. E_a is the machine back emf. Furthermore, for the machine state equations, the armature winding loop equation:

$$V_a = R_a i_a + L_a\left(\frac{di_a}{dt}\right) + \left(\frac{N_a}{N_f}\right)\left(\frac{d\Phi_f}{dt}\right) + k_m \Phi_f \omega_f \tag{1}$$

In eq. (1) and for separately excited machines, the non-linear magnetizing characteristic is given by:

$$\Phi_f = \left(\frac{c_1 i_f}{c_2 + c_3 i_f + c_4 i_f^2}\right)$$

Furthermore in eq. (1), the inverse magnetizing characteristic is expressed as:

$$I_f = \left(\frac{c_1 - c_3\Phi_f - \sqrt{(c_1 - c_3\Phi_f)^2 - 4c_2c_4\Phi_f^2}}{2c_2\Phi_f}\right)$$

and the field winding loop equation:

$$V_f = R_f\left(\frac{c_1 - c_3\Phi_f - \sqrt{(c_1 - c_3\Phi_f)^2 - 4c_2c_4\Phi_f^2}}{2c_2\Phi_f}\right) + \left(\frac{d\Phi_f}{dt}\right) \tag{2}$$

Expressing nonlinear equation of the machine shaft motion by:

$$J\left(\frac{d\omega_r}{dt}\right) = (k_m \Phi_f i_a) - (T_{load}) \tag{3}$$

In general context, machine dynamics are represented by a linear state-space:

Linear model: $\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{w}(t)$

Nonlinear state model: $\dot{\boldsymbol{x}}(t) = f(\boldsymbol{x}(t)) + g(\boldsymbol{x}(t))\boldsymbol{u}(t) + \boldsymbol{w}(t)$ (4)

The machine states, (*input*) and (*unknown*) disturbances are thus defined by:

Machine state $\boldsymbol{x}(t) = (x_1(t) \quad x_2(t) \ldots x_n(t))^t \in \Re^{n\times 1}$

machine input $\boldsymbol{u}(t) = (u_1(t) \quad u_2(t) \ldots u_n(t))^t \in \Re^{m\times 1}$

machine disturbance $\boldsymbol{w}(t) = (w_1(t) \quad w_2(t) \ldots w_n(t))^t \in \Re^{n\times 1}$ (5)

It is assumed that, the machine is subjected to disturbances, $\boldsymbol{w}(t)$. Disturbances are acting at output rotating shaft with an upper bound $w_{ub} \geq \|w(t)\|$.

2.2. Building T-S linear models out of nonlinear motor models

In reference to fig. 2., building fuzzy models for dynamic systems, have been suggested by Takagi and Sugeno (T-S), as in Massoud and Yazdanpanah [7]. T-S model statement is used to represent local linear input-output relations. Once the machine is represented by a (T-S) model, it is described by *if-then* fuzzy rules. The i^{th} (T-S) fuzzy rule of is stated as:

Rule i: *if* ($z_1(t)$ *is* M_{i1} ... *and* $z_g(t)$ *is* M_{ig}) ($i = 1,2,\ldots,r$) *then* ($\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{B}_i\boldsymbol{u}(t) + \boldsymbol{w}(t)$)

Machine output:

Rule i: *if* ($z_1(t)$ *is* M_{i1} ... *and* $z_g(t)$ *is* M_{ig}) ($i = 1,2,\ldots,r$) *then* $\mathbf{y}(t) = \mathbf{C}_i\mathbf{x}(t)$ (6)

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where,

$$\boldsymbol{z}(t) = \left(z_1(t),\quad z_2(t), \ldots z_n(t)\right)^T \in \Re^{n\times 1} \tag{7}$$

$$\boldsymbol{A}_i \in \Re^{n\times n}, \boldsymbol{B}_i \in \Re^{n\times m}, \boldsymbol{C}_i \in \Re^{o\times n} \tag{8}$$

(r) in eq. (6) is the number of if-then rules, M_{ig} is fuzzy membership associated with i^{th} rule and g^{th} parameter component. There are two functions of $\boldsymbol{z}(t)$ with each rule. The first is a degree of fulfillment i^{th} rule as:

$$\mu_i(\boldsymbol{z}(t)) = \prod_{j=1}^{g} M_{ij}\left(z_j(t)\right) \tag{9}$$

In eq. (9), $M_{ij}\left(z_j(t)\right)$ is grade of membership of $z_j(t)$. The possibility that i^{th} rule fires, is given by product of all membership functions associated with i^{th} rule. All μ_i are non-negative functions and truth value of at least one rule is nonzero. A firing probability for an i^{th} rule is defined by:

$$h_i(\boldsymbol{z}(t)) = \frac{\mu_i(\boldsymbol{z}(t))}{\sum_{i=1}^{r} \mu_i(\boldsymbol{z}(t))} \quad (0 \to 1) \tag{10}$$

In eq. (9), it is assumed that:

$$\mu_i(\boldsymbol{z}(t)) \geq 0 \quad \text{and} \quad \sum_{i=1}^{r} \mu_i(\boldsymbol{z}(t)) > 0 \tag{11}$$

for all t. Therefore, we get,

$$h_i(\boldsymbol{z}(t)) \geq 0 \tag{12}$$

$$\sum_{i=1}^{r} h_i(\boldsymbol{z}(t)) = 1 \tag{13}$$

Using the center of gravity for defuzzification, output of a T-S fuzzy system is finally expressed:

$$\dot{\boldsymbol{x}}(t) = \frac{\sum_{i=1}^{r} \mu_i(\boldsymbol{z}(t))\left(\boldsymbol{A}_i \boldsymbol{x}(t) + \boldsymbol{B}_i \boldsymbol{u}(t)\right)}{\sum_{i=1}^{r} \mu_i(\boldsymbol{z}(t))} + \boldsymbol{w}(t) \tag{14}$$

$$\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{r} h_i(\boldsymbol{z}(t))\left(\boldsymbol{A}_i \boldsymbol{x}(t) + \boldsymbol{B}_i \boldsymbol{u}(t)\right) + \boldsymbol{w}(t) \tag{15}$$

$$\boldsymbol{y} = \sum_{i=1}^{r} h_i(\boldsymbol{z}(t)) \boldsymbol{C}_i \boldsymbol{x} \tag{16}$$

Fuzzy linear state models are hence given by eq. (15) and eq. (16).

2.3. Fuzzy (T-s) machine models and stability condition

It is needed to evaluate control gains $\boldsymbol{K}_j$ for the machine fuzzy controller, while guaranteeing closed-loop stability. Once $\boldsymbol{u}(t) = 0$, machine fuzzy open loop description of eq. (15) is restated as follows:

$$\dot{\boldsymbol{x}}(t) = \frac{\sum_{i=1}^{r} \mu_i(\boldsymbol{z}(t))\left(\boldsymbol{A}_i \boldsymbol{x}(t)\right)}{\sum_{i=1}^{r} \mu_i(\boldsymbol{z}(t))}$$

$$\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{r} h_i(\boldsymbol{z}(t))\left(\boldsymbol{A}_i \boldsymbol{x}(t)\right) \tag{17}$$

Linear consequent equations represented by $\boldsymbol{A}_i \boldsymbol{x}(t)$ is a sub-system. Sub-systems are asymptotically stable if there exists a common positive definite matrix $\boldsymbol{P}$:

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$$\left(A_i^T P + PA_i < 0\right) \tag{18}$$

Eq. (18) depends on motor sub-models, i.e. A_i it does not depend on the disturbance $w(t)$. This is reduced to a Lyapunov stability definition for linear systems if $(r = 1)$. Finding a common Lyapunov function P for eq. (18) can be solved by convex programming algorithm. This involves using LMI's. From eq. (2) and eq. (15), the approximation error between the nonlinear machine eq. (2) and the fuzzy model eq. (15) is expressed:

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t) + w(t)$$

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))[A_i x(t) + B_i u(t)] + \left\{\left(f(x) - \sum_{i=1}^{r} h_i(z(t))[A_i x(t)]\right) + \left(g(x) - \sum_{i=1}^{r} h_i(z(t))(B_i)\right)u(t)\right\} + w(t) \tag{19}$$

And $$\left\{\left(f(x) - \sum_{i=1}^{r} h_i(z(t))[A_i x(t)]\right) + \left(g(x) - \sum_{i=1}^{r} h_i(z(t))(B_i)\right)u(t)\right\} \tag{20}$$

2.4. Machine fuzzy controller law

A state feedback fuzzy controller of $u(t) = K_i x(t)$ is employed to deal with the control system design. In terms of fuzzy rules, this is given by:

j^{th} Control Rule:

Rule j: *if $z_1(t)$ is M_{j1} … and $z_g(t)$ is M_{jg} then* $u(t) = K_i x(t)$ (21)

For $(j = 1, 2, \ldots, r)$, the overall fuzzy controller is defined by:

$$u(t) = \frac{\sum_{j=1}^{r} \mu_j(z(t))(K_j x(t))}{\sum_{j=1}^{r} \mu_j(z(t))} \tag{22}$$

$$u(t) = \sum_{j=1}^{r} h_j(z(t))(K_j x(t)) \tag{23}$$

$h_j(z(t))$ was defined by eq. (10) and K_j are the control parameters for j from $(1 \to r)$.

2.5. Closed loop of fuzzy modeled machine system

To acquire entire closed loop machine dynamics, eq. (23) is substituted into eq. (19). This yields:

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t) + w(t)$$

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))(A_i + B_i K_j)\, x(t) + \left(f(x) - \sum_{i=1}^{r} h_i(z(t))A_i x(t)\right)$$
$$+ \sum_{i=1}^{r} h_i(z(t))\sum_{j=1}^{r} h_j(z(t))(g(x) - B_i)K_j x(t) + w(t)$$

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))(A_i + B_i K_j)\, x(t) + \Delta f + \Delta g + w(t) \tag{24}$$

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In eq. 24, Δf and Δg are errors between fuzzy model and the actual machine model. They are expressed as:

$$\Delta f = \left(f(\boldsymbol{x}) - \sum_{i=1}^{r} h_i(\boldsymbol{z}(t)) \boldsymbol{A}_i \boldsymbol{x}(t) \right) \tag{25}$$

$$\Delta g = \sum_{i=1}^{r} h_i(\boldsymbol{z}(t)) \sum_{j=1}^{r} h_j(\boldsymbol{z}(t)) (g(\boldsymbol{x}) - \boldsymbol{B}_i) \boldsymbol{K}_j \boldsymbol{x}(t) \tag{26}$$

3. A fuzzy based controller synthesis with an H_∞ character

Disturbance rejection is an important character while designing controllers. We shall assume that $\boldsymbol{w}(t)$ is an unknown but bounded disturbance. The effect of $\boldsymbol{w}(t)$, does deteriorate the control performance of fuzzy control. Therefore, how to eliminate the effect of $\boldsymbol{w}(t)$ to guarantee the control performance is the controller objective. Since H_∞ control is the most important control design to efficiently eliminate the effect of $\boldsymbol{w}(t)$, it will be employed to deal with the robust performance control. Considering the following H_∞ control performance:

$$\left(\frac{\int_0^{t_f} \boldsymbol{x}^T(t) \boldsymbol{Q} \boldsymbol{x}(t) dt}{\int_0^{t_f} \boldsymbol{w}^T(t) \boldsymbol{w}(t) dt} \right) < \gamma^2 \tag{27}$$

Or even it can expressed as:

$$\int_0^{t_f} \boldsymbol{x}^T(t) \boldsymbol{Q} \boldsymbol{x}(t) dt < \gamma^2 \int_0^{t_f} \boldsymbol{w}^T(t) \boldsymbol{w}(t) dt \tag{28}$$

In eq. (28), t^f denotes the terminal time of the control, γ is a prescribed value which denotes the worst effect of $\boldsymbol{w}(t)$ on $\boldsymbol{x}(t)$, and $\boldsymbol{Q}$ is a positive-definite weighting matrix. A physical meaning of eq. (28) is that, effect of the disturbance $\boldsymbol{w}(t)$ must be attenuated below a desired level γ from energy point of view. A desired level of γ is chosen as a positive small value less than unity for attenuation of $\boldsymbol{w}(t)$. Inequality given by eq. (27) can be seen as bounded disturbance and bounded state but with a prescribed gain γ. If the initial condition is also considered, the inequality eq. (35) is modified as :

$$\int_0^{t_f} \boldsymbol{x}^T(t) \boldsymbol{Q} \boldsymbol{x}(t) dt < \boldsymbol{x}^T(0) \boldsymbol{P} \boldsymbol{x}(0) + \gamma^2 \int_0^{t_f} \boldsymbol{w}^T(t) \boldsymbol{w}(t) dt \tag{29}$$

In eq. (29), $\boldsymbol{P}$ is some symmetric positive-definite weighting function. The design purpose of a fuzzy control system is to specify a linear fuzzy controller such that both the stability of fuzzy linear control and the H_∞ control performance in eq. (28) with a prescribed attenuation level γ are guaranteed. The robustness optimization is to achieve a minimum γ^2 in eq. (28) to obtain maximum elimination of the effect of the disturbance $\boldsymbol{w}(t)$. For the nonlinear motor system eq. (2), this design problem is reduced to identify a stabilizable fuzzy control K as will be discussed in the coming section.

3.1. H_∞ control design, (Lyapunov approach)

The design purpose is to specify a fuzzy linear control law given in eq. (23) for the nonlinear system in eq. (24) with a guaranteed H_∞ performance in eq. (28). Since the system in eq. (24) is nonlinear system, then we shall be choosing a LYAPUNOV function of :

$$V(t) = \boldsymbol{x}^T(t) \boldsymbol{P} \boldsymbol{x}(t) \tag{30}$$

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The weighting matrix $\boldsymbol{P}$ is, $\left(\boldsymbol{P}=\boldsymbol{P}^T>0\right)$. Time derivative of $V(t)$ is:

$$\dot{V}(t)=\dot{\boldsymbol{x}}^T(t)\boldsymbol{P}\boldsymbol{x}(t)+\boldsymbol{x}^T(t)\boldsymbol{P}\dot{\boldsymbol{x}}(t) \tag{31}$$

For $\dot{\mathbf{x}}(t)$ in eq. (31), substituting eq. (24), once the fuzzy controller eq. (23) is employed in eq. (2), there exists a positive-definite matrix $\left(\boldsymbol{P}=\boldsymbol{P}^T>0\right)$ such that the following matrix inequalities :

$$\boldsymbol{A}_i^T\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A}_i+\boldsymbol{P}\boldsymbol{B}_i\boldsymbol{K}_j+\boldsymbol{K}_j^T\boldsymbol{B}_i^T\boldsymbol{P}+\boldsymbol{A}_p^T\boldsymbol{A}_p+\left(\boldsymbol{B}_p\boldsymbol{K}_j\right)^T\left(\boldsymbol{B}_p\boldsymbol{K}_j\right)+\left(2+\frac{1}{\gamma^2}\right)\boldsymbol{P}\boldsymbol{P}+\boldsymbol{Q}<0 \tag{32}$$

are satisfied for $i, j=1,2,\ldots, r$. The closed-loop system of eq. (24) is bounded and H^∞ performance of eq. (29) is satisfied, where $c_1=\lambda_{min}(\boldsymbol{Q})$, whenever $\|\boldsymbol{x}(t)\|>\gamma w_{bd}/\sqrt{c_1}$, and $\dot{V}(t)<0$. Using LYAPUNOV extension, this demonstrates that the trajectories of the closed-loop system eq. (24) are bounded, resulting for $t=0$ to $t=t_f$, in:

$$V(t_f)-V(0)<-\int_0^{t_f}\boldsymbol{x}^T(t)\,\boldsymbol{Q}\,\boldsymbol{x}(t)dt+\gamma^2\int_0^{t_f}\boldsymbol{w}^T(t)\boldsymbol{w}(t)dt \tag{33}$$

$$\int_0^{t_f}\boldsymbol{x}^T(t)\,\boldsymbol{Q}\,\boldsymbol{x}(t)dt<\boldsymbol{x}^T(0)\boldsymbol{P}\boldsymbol{x}(0)+\gamma^2\int_0^{t_f}\boldsymbol{w}^T(t)\boldsymbol{w}(t)dt \tag{34}$$

this is eq. (29) and the H_∞ control performance is achieved with a prescribed γ^2. In the case of $\boldsymbol{w}(t)$, if the fuzzy controller eq. (24) is employed in the closed-loop nonlinear system eq. (25) and there exists a positive-definite matrix $\left(\boldsymbol{P}=\boldsymbol{P}^T>0\right)$ such that the matrix inequalities given in eq. (32) are valid. This results in quadratically stable closed-loop system of eq. (24).

4. H_∞ Linear matrix inequality, LMI formulation

It is not obvious to find an analytical common solution in such away $\left(\boldsymbol{P}=\boldsymbol{P}^T>0\right)$ for eq. (32). The synthesis problem is reformulated into the Linear Matrix Inequality problem (LMI). The matrix inequalities in eq. (32) are transformed to the equivalent LMI's by introducing new variables $\boldsymbol{W}$ and $\boldsymbol{Y}_j$, in such a way $\left(\boldsymbol{W}=\boldsymbol{P}^{-1}\right)$ and $\left(\boldsymbol{Y}_j=\boldsymbol{K}_j\boldsymbol{W}\right)$, is equivalent to the following matrix inequalities:

$$\boldsymbol{W}\boldsymbol{A}_i^T+\boldsymbol{A}_i\boldsymbol{W}+\boldsymbol{B}_i\boldsymbol{Y}_j+\boldsymbol{Y}_j^T\boldsymbol{B}_i^T+\boldsymbol{W}\boldsymbol{A}_p^T\boldsymbol{A}_p\boldsymbol{W}++\left(\boldsymbol{B}_p\boldsymbol{Y}_j\right)^T\left(\boldsymbol{B}_p\boldsymbol{Y}_j\right)+\left(2+\frac{1}{\gamma^2}\right)\boldsymbol{I}+\boldsymbol{W}\boldsymbol{Q}\boldsymbol{W}<0 \tag{35}$$

Expressed in matrix form:

$$\begin{pmatrix} \boldsymbol{W}\boldsymbol{A}_i^T+\boldsymbol{A}_i\boldsymbol{W}+\boldsymbol{B}_i\boldsymbol{Y}_j+\boldsymbol{Y}_j^T\boldsymbol{B}_i^T+\left(2+\frac{1}{\gamma^2}\right)\boldsymbol{I} & \left(\boldsymbol{B}_p\boldsymbol{Y}_j\right)^T & \boldsymbol{W} \\ \boldsymbol{B}_p\boldsymbol{Y}_j & -\boldsymbol{I} & 0 \\ \boldsymbol{W} & 0 & -\left\{\boldsymbol{A}_p^T\boldsymbol{A}_p+\boldsymbol{Q}\right\}^{-1} \end{pmatrix}<0 \tag{36}$$

for $(i,j=1,2,\ldots,r)$. If the LMI's in eq. (35) have a positive-definite solution for $\boldsymbol{W}$, the closed-loop system is stable and the H_∞ control performance in eq. (29) is guaranteed for a γ. Finally, the H_∞ optimization design for fuzzy control system of eq. (2) is formulated as the following constrained optimization problem:

$$Minimize\ \gamma^2 \quad \text{Subject to} \quad \left(\boldsymbol{W}=\boldsymbol{W}^T>0\right)$$

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and

$$\begin{pmatrix} \mathbf{W}\mathbf{A}_i^T + \mathbf{A}_i\mathbf{W} + \mathbf{B}_i\mathbf{Y}_j + \mathbf{Y}_j^T\mathbf{B}_i^T + \left(2+\frac{1}{\gamma^2}\right)\mathbf{I} & (\mathbf{B}_p\mathbf{Y}_j)^T & \mathbf{W} \\ \mathbf{B}_p\mathbf{Y}_j & -\mathbf{I} & 0 \\ \mathbf{W} & 0 & -\left\{\mathbf{A}_p^T\mathbf{A}_p + \mathbf{Q}\right\}^{-1} \end{pmatrix} < 0 \tag{37}$$

Solutions for $\mathbf{W}$ and $\mathbf{Y}_j$ are computed numerically by convex optimization algorithm. This is done in reference to the LMI Toolbox of Matlab. By expressing eq. (37) in to the standard form, H_∞ controller $\mathbf{K}_j$ is synthesized. Having derived an LMI formulation, the next section shows how to validate the proposed fuzzy control methodology.

5. Results, Neuro-Fuzzy model building and controller synthesis

5.1 Machine dynamics validation

Within this section we shall present few simulation results. We shall adopt the following machine real parameters. Armature winding resistance $R_a = 2.85\Omega$, armature winding inductance $L_a = 0.008H$, field winding resistance $R_f = 960\Omega$, field winding voltage $V_f = 240V$, armature winding turns: $N_a = 50$, field winding turns $N_f = 1040$. Torque design constant $R_a = 2.85\Omega$, $k_m = 1.713Nm/WbxA$. In addition, field residual flux $\Phi f_r = 0.0014WbT$, and shaft inertia $J = 0.0088kgm^2$. The nonlinear magnetizing characteristic of the machine is given by eq. (1), and V_a is the armature winding input voltage. In fig. 3. initially, we verified the machine dynamic and motion as were described by eq. (1). A typical machine speed versus time is shown, where a realistic machine behavior with typical machine parameters were obtained. In fig 4., learning patterns generation through suitable machine excitation are shown. In this respect, fig. 4(a) presents the randomly input excitations of voltage to the machine armature winding, whereas in fig. 4(b) we show another random excitations of load torque T_{load}, as input to machine outer shaft. After learning, we shall extract the linear fuzzy models.

5.2. Building of machine linear models via the five layers Neuro-Fuzzy system

A fuzzy model can be constructed from (I/O) training patterns. In this respect, we have already shown the corresponding I/O training data used for the modeling. Two inputs (voltage and input torques, and one output, the motor speed) are shown. Furthermore, we have shown the results of modeling the nonlinear machine. It shows that, although the motor has a highly nonlinear behavior, but the used fuzzy modeling algorithm was able to follow the motor actual response. The modeling error is small. This shows how accurate the fuzzy linear models are. Such linearized modes will be shortly used in designing a robust controller via the LMI. A typical corresponding state space motor sub-models are:

$$A_1 = \begin{pmatrix} 1.5215 & -0.527 & 0.0004 & -0.0007 \\ 1.00 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1.00 & 0 \end{pmatrix} \quad A_2 = \begin{pmatrix} 1.546 & -0.55 & 0.0002 & -0.0002 \\ 1.00 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1.00 & 0 \end{pmatrix}$$

$$B_1 = \begin{pmatrix} 0.0008 & 0 \\ 0 & 0 \\ 1.00 & 0 \\ 0 & 0 \end{pmatrix} \quad B_2 = \begin{pmatrix} 0.0011 & 0 \\ 0 & 0 \\ 1.00 & 0 \\ 0 & 0 \end{pmatrix} \quad C_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} \quad C_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}$$

Two linear sub-models state-space matrices have been derived for the machine dynamics. Neuro-fuzzy modeling is applied to the problem of identifying a discrete machine model. A fuzzy model can

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be constructed from data by using outputs of the clustering algorithm and by constructing regressors to form inputs to the used Neuro-Fuzzy architecture.

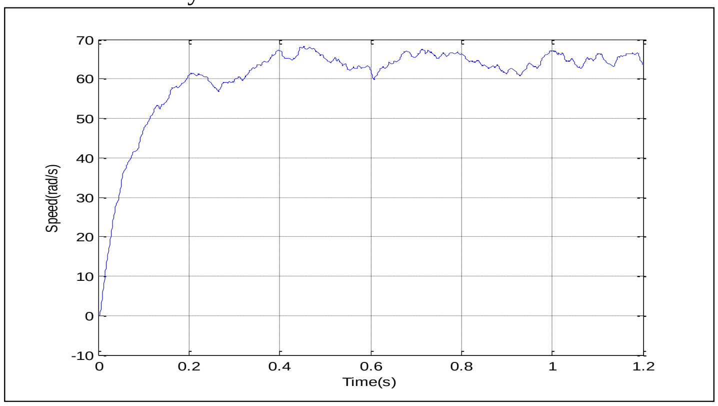
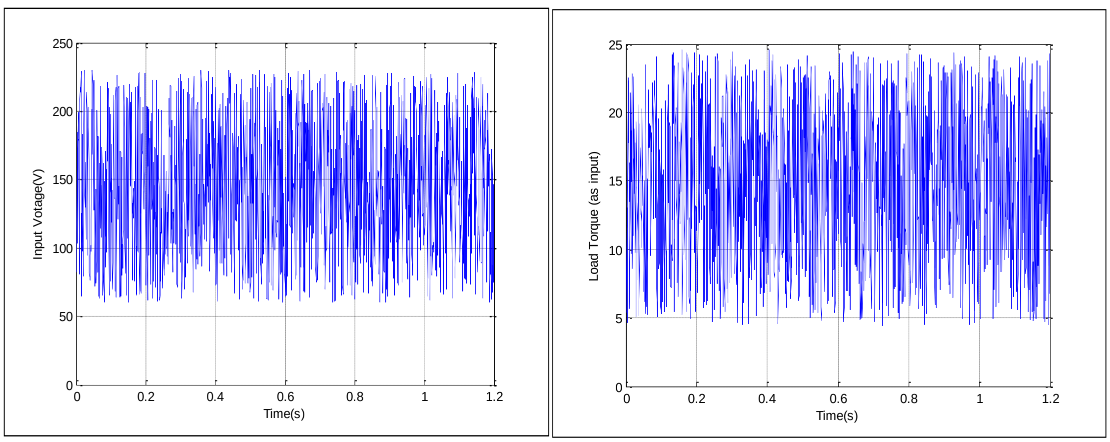


Figure 3. Initially, verifying the machine dynamic and motion as described by eq. (1).
A typical machine speed versus time. A realistic machine behavior with physical machine parameters.



(*a*) (*b*)

Figure 4. Learning patterns generation through suitable machine excitation.
(*a*) Random input excitations of voltage to the machine armature winding.
(*b*) Another random excitations of load torque, as input to machine outer shaft.

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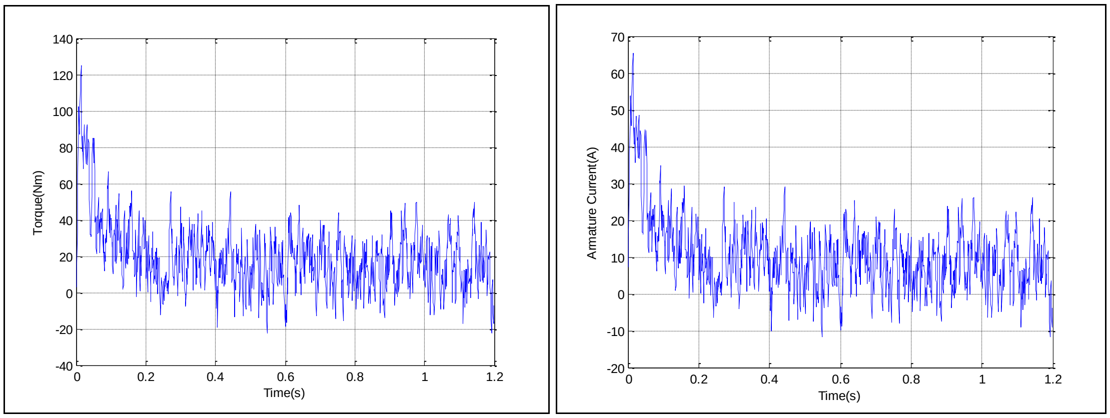
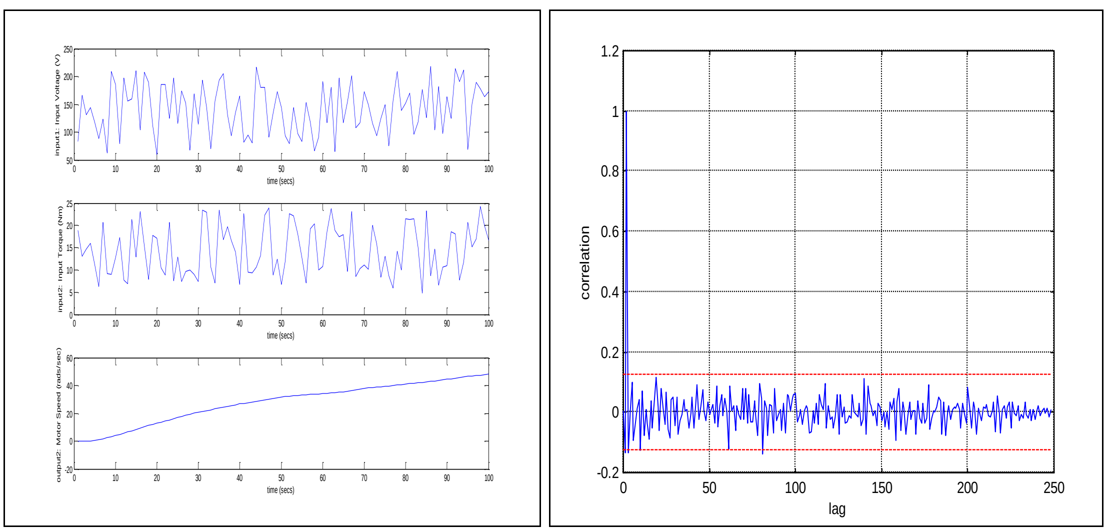


(*a*) (*b*)

Figure 5. Learning pattern gathering through tabulation of machine responses.
(*a*) Machine electromagnetic torque (*Nm*) versus time.
(*b*) Machine armature current in (*A*) versus time.



(*a*) (*b*)

Figure 6. Validating the resulting fuzzy sub-models.
(*a*) Typical machine (I/O) training patterns.
(*b*) Modeling error autocorrelation of the machine system. Autocorrelation figures indicate that, the Neuro-fuzzy has produced high degree of modeling accuracy.

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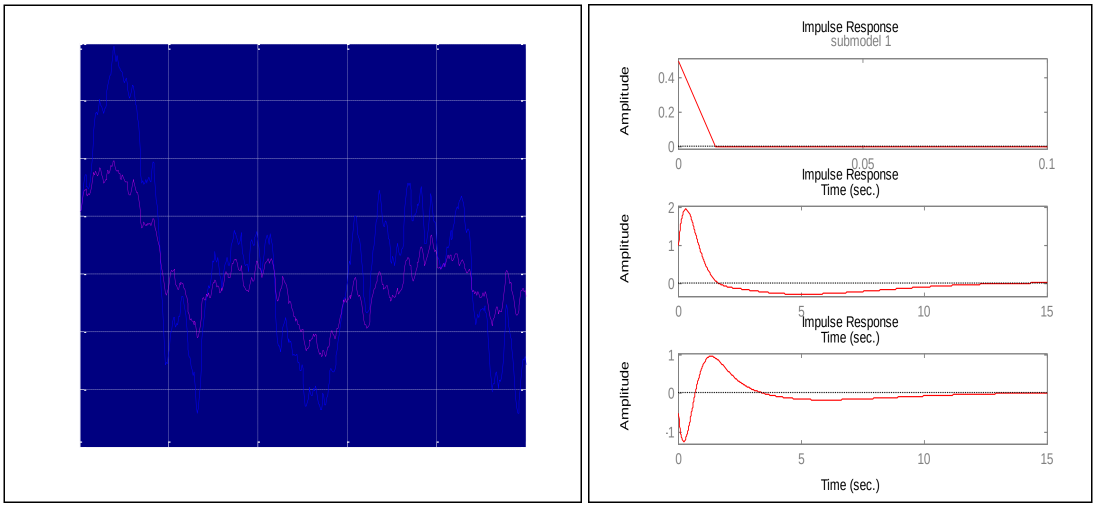

(a) (b)

Figure 7. Validating machine open and closed loop responses.

(a) Open loop machine actual real-time output response, compared to Neuro-Fuzzy model response. This is used for validating the Neuro-Fuzzy model building.

(b) Different machine closed loop impulse output responses. Controllers $\boldsymbol{K}_j$ were synthesized for various machine operating regions, while relying on LMI approach.

5.3 Training patterns generation, gathering, and fuzzy linear model validation

In section (4) we have presented a methodology of synthesizing a robust fuzzy based H_∞ controller for the nonlinear electrical machine system. That was based in using the LMI formulation given by eq. (37). This requires the gathering of linear fuzzy sub-models via training the adopted five layers Neuro-Fuzzy system. Within this context, we shall now rather focus on simulation results for such a typical H_∞ machine controller synthesis.

In this respect, fig. 5 shows the gathering of learning patterns, through tabulation of machine responses. In this particular regard, fig. 5(a) is the resulting machine electromagnetic torque versus time, whereas, fig. 5(b) is the machine armature current versus time. At this stage, the five layers Neuro-Fuzzy is ready to be trained and learn the linear fuzzy machine models.

For Validating the synthesized Fuzzy Models, in fig. 6., we show typical obtained results for validating the resulting fuzzy sub-models. In fig. 6(a), we show particular typical machine (I/O) training patterns. The training patterns were selected in such a way that to let the machine to be excited within the most possible frequencies of operational spectrum. In corresponding to this, in fig. 6(b) we likewise present the modeling error autocorrelation of the machine system. Autocorrelation results are indicating that, the Neuro-fuzzy has produced high degree of modeling accuracy. Neuro-fuzzy has produced high degree of accuracy. Large number of errors are located near to the zeros with (425). This shows a great deal of modeling accuracy. A conventional linear difference model with REGRESSORS is constructed, hence containing previous inputs and outputs. The created machine model has six inputs and two outputs, hence two groups of seven sets of MFs are shown. Each universe of discourse (set) has three MFs representing the assigned three clusters. Such memberships are representing the inputs range.

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5.4 Training patterns generation, gathering, and fuzzy linear model validation

Finally, in fig. 7. we do a validation of the machine open and closed loop responses, while incorporating the obtained linear fuzzy sub models. In this respect, in fig. 7(a) we show machine real-time output responses, compared to Neuro-fuzzy model response, hence validating the linear outputs of the Neuro-fuzzy model building.

In fig. 7(a) we show the open loop machine actual real-time response, compared to FN model response, validating the Neuro-fuzzy model building. Furthermore, fig. 7(b), we show the closed loop machine responses via controller synthesis for various operating machine regions. It is very clear that, the synthesized linear fuzzy controllers has produced good degree of accuracy in terms of the desired H_∞ performance.

The figure of fig. 7(b), is also a clear evidence that the controller synthesis for various operating machine regions, is resulting excellent machine return to original state. This verifies the controller synthesis, while relying on fuzzy sub-linear models of the machine.

5.5 Results of machine behavior with Neuro-Fuzzy models

Further analysis of the results given in fig. 7(b), it shows how the synthesized controller is able to robustly regulate the electrical machine output. It indicates the machine is being brought back to the zero state, thought after the system has been subjected to a very worse disturbance at its input. Feedback gains are used to form a closed loop system. Further examination of the closed loop frequency responses over a quit wide range of frequencies has revealed that the motor controller system was able to reject the effect of disturbances with high frequencies components. The three responses are for machine outputs over the time domain are shown in fig. 7 (b). The response is settled down of maximum of (11 seconds), and a minima of (0.02 seconds). This is considered as a fast settling time according to the machine system subject to the worst disturbance effect.

6. Conclusion

In this manuscript a study regarding a modeling of a nonlinear electrical machine has been presented. The approach followed was related to the use of some kind of intelligent modeling technique known as the TAKAGI AND SUGENO (T-S). T-S modeling has been employed to extract some sub-linear fuzzy models of a nonlinear electric machine system. The system under study was excited with some sort of random input signals, hence, the corresponding outputs were recorded. Input-output pattern did represent the training data for the proposed fuzzy system. After the machine sub-models were acquired, robust sub controllers were designed accordingly via an LMI approach. The employed Neuro-Fuzzy system was an excellent approach in terms of building linear models. In addition, the presented Neuro-Fuzzy was a transparent approach in terms building linear fuzzy models, not similar to the approach of Neural Network, where it is considered as a black box modeling approach.

However, the presented Neuro-Fuzzy has a number of limitations. The primary one is related to the training and learning time. As the network architecture gets complicated, the training and learning rates get much difficult to achieve. Therefore, the design of such a Neuro-Fuzzy, is a tradeoff between network size, and the modeling accuracy. Another related issue to Neuro-Fuzzy limitation, is related to the smoothness in while presenting the training patterns. It was found, once the training patterns are totally not smooth, the Neuro-Fuzzy was finding it difficult in ending the training phase.

The presented work will be further expanded. This involves to train the Neuro-Fuzzy for further training samples, with even worst operational behavior of the machine. The additional will involve the

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expanding of the network number of neurons, within each layer. This will result in the additional fuzzy *if-then* rules. The additional work will also involve reengineering of the control system architecture, in ach away that looks as a reference based control system, rather than a regulator class control.

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